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SENSOR INDEPENDENT COMPLEX DATA (SICD)

Volume 3

Image Projections Description Document

Exploitation processing description for SICD data products.
Projections from Image To Scene & Scene To Image.

(2018-12-13)

Version 1.2.1

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FOREWORD

The suite of Sensor Independent Complex Data (SICD) standardization documents describe the implementation of SICD products for complex image data generated by Synthetic Aperture Radar (SAR) systems and their data processing elements.

A SAR complex image is an intermediate data product. The real utility is in the products and measurements that may be derived from it. The quality of the pixel array (resolution, SNR, etc.), along with the set of metadata provided, are critical in generating the derived products. The "sensor independence" of the SICD product refers to the ability of the allowed pixel array and metadata options to accurately describe the image products from many sensors and data processing systems. Sensor independence does NOT mean that all products have the same format for the pixel array or the same set of metadata parameters.

The SICD documentation has been organized into three volumes and a set of XML implementation artifacts. The three volumes are summarized below. The collection of SICD XML artifacts includes the schema documents that define the correct implementation of the XML metadata document included in a given product.

Volume 1 Design & Implementation Description Document

Contains the description needed by producers of SAR complex image products to design a SICD product and the set of metadata that describe it.

Volume 2 File Format Description Document

Defines the placement of SICD data products in the allowed image file formats. Also provides the description needed by users of SICD products to read and properly extract the SICD data components from a SICD product file.

Volume 3 Image Projections Description Document

Describes the SICD sensor model and the correct projections from image location to ground point and from ground point to image location for all SICD products.

A companion suite of standardization documents, collectively known as Sensor Independent Derived Data (SIDD), describes standardized products and measurements that may be derived from SICD.

The SICD and SIDD documentation and associated XML artifacts are available on the National System for Geospatial-Intelligence (NSG) Standards Registry (https://nsgreg.nga.mil).

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1 Introduction

The mapping of SAR image pixel locations to geolocated ground points is a fundamental step in the exploitation of the image. The SAR image pixel grid locations represent a two-dimensional projection of the three-dimensional scene that was imaged. The projection is a function of the collection geometry and the image formation processing. The projection from three-dimensional scene points to two-dimensional image grid locations are along contours of constant range and constant range rate. All points in scene that lie along a contour will have their image signal response centered at the same image location.

The mapping of image pixel grid locations to geolocated points in the imaged scene involves two fundamental projection computations. See Figure 1-1.

(1) Image To Scene: The projection from image pixel grid location to a geolocated point in the scene that was imaged. A fixed point scatterer located at the point in the scene has its two-dimensional image impulse response centered on the image grid location.

(2) Scene To Image: The projection from geolocated point in the scene to the image pixel grid location. The image pixel grid location is the center of the two-dimensional image signal response of a fixed point scatterer located at the scene point.

The following sections provide a description of these computations for SICD image products using the metadata supplied with the product. For all computations, the image pixel grid is considered to be a continuous two-dimensional space. Image pixel grid locations expressed in image pixel indices (irow,icol) may include fractional values. See Section 2.2.

The Image To Scene projection maps an image grid location to a geolocated surface. The geolocated surface is most commonly a ground plane but may be any of a number of more complex surfaces. For all image products, the image grid location uniquely determines a contour of constant range and constant range rate (referred to as an “R/Rdot” contour). The R/Rdot contour is relative to the Aperture Reference Point (ARP) position and velocity at the Center Of Aperture time (t_{COA}) for the grid location. The intersection of the R/Rdot contour with the geolocated surface establishes the three-dimensional scene point position. Only intersection points on the correct Side of Track (either left side or right side) are considered. For simple surfaces such as a ground plane, the R/Rdot contour will have a single intersection and yield a single scene point. For complex surfaces such as a terrain height model of a mountainous scene, the R/Rdot contour may yield multiple intersection points. The multiple scene points may yield an ambiguity in the precise scene location.

The Scene To Image projection maps a geolocated point in the three-dimensional scene to a single image grid location. Only scene points on the correct Side of Track yield meaningful image grid locations. The projection computation finds the image grid location for which the R/Rdot projection contour passes through the scene point. In general, a precise computation requires an iterative approach. The iterative approach computes an initial image grid location estimate. For the estimated grid location, the precise R/Rdot contour is computed and

projected to a ground plane that contains the scene point. The displacement between the scene point and the R/Rdot contour/ground plane point of intersection is compared to a user selected accuracy threshold. For acceptably small displacements (e.g. less than a fraction of the image resolution), the estimated image location is accepted. For larger than acceptable displacements, the displacement is used to compute a refined estimate of the image grid location. The iterative approach presented is applicable for all image products.

The Image To Scene and Scene To Image projections do not place any restrictions on the geolocated surface used. Only the portion of the surface on the correct Side of Track and at ranges and angles consistent with the imaging collection and processing are of interest. Also, the surface considered may or may not have been used explicitly in the image formation processing.

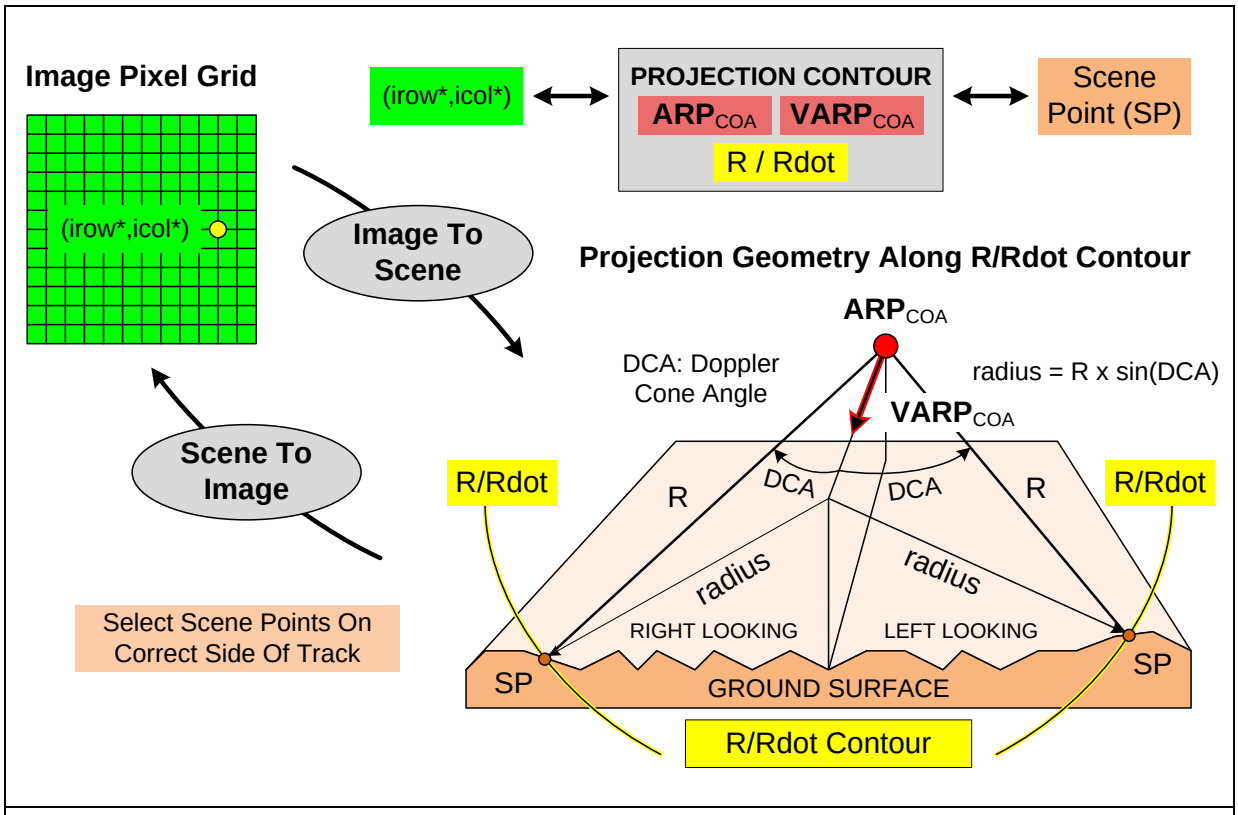


Figure 1-1 Image To Scene & Scene To Image
Image grid location (irow*, icol*) projects to the ground surface along a R/Rdot contour.

1.1 Ground Plane Projections: Precise Versus Simple Projection

A common application of the Image to Scene and Scene To Image projections is the resampling of the complex image grid to an evenly spaced grid in a ground plane. For each ground plane grid location, the image pixel grid location is computed. The computed image grid location will, in general, not lie precisely on a pixel center. The image signal at the grid

location is then computed (via a user selected image interpolation method). The interpolated image signal value is then assigned to the ground plane grid location.

The precise resampling from image grid to ground plane grid can be computationally intensive. For each ground plane location, the image grid location is (for many images) an iterative computation where multiple image grid estimates are computed. For each estimated image grid location, the precise R/Rdot contour is computed and then projected back to the ground plane. The image grid to R/Rdot contour computation is specific to the image grid type and the image formation algorithm. For large images or limited compute power, precise resampling can be an excessively time consuming computational task.

For many applications, such as generating an overview image for viewing, a less precise but greatly simplified projection algorithm is adequate. A simple projection method is presented that is computationally fast and can be applied to any SICD image product. See Figure 1-2. The projection is precise for the SCP pixel, located near the center of the image, to the ground plane. Absolute accuracy degrades slowly for pixels displaced from the SCP pixel. The simple projection method approximates the image pixel grid as a uniformly spaced grid in the image plane. Ground plane points are projected along straight lines to the image plane to compute the image grid locations. The simple projection is described in Section 7.

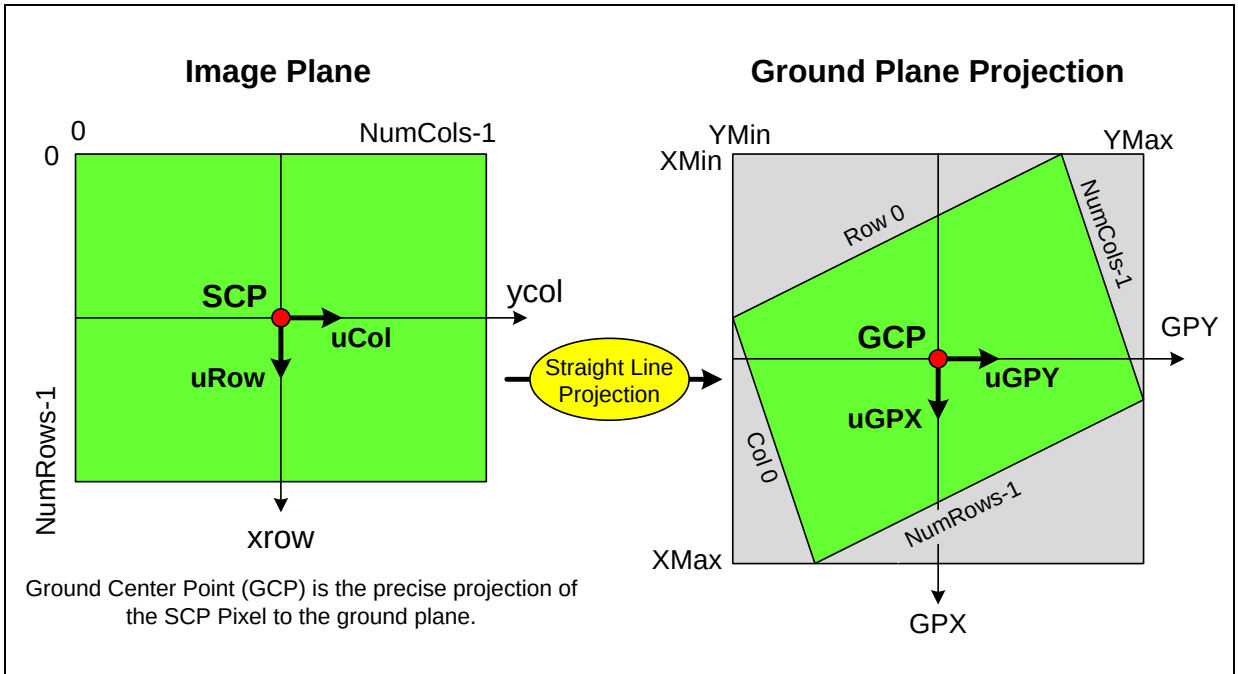


Figure 1-2 Simple Image To Ground Plane Projection
The projection from image plane to ground plane is along straight lines.

1.2 SICD Metadata Parameter List

The image projection computations described in the following sections use the metadata parameters listed in Table 1-1. For each parameter, the parameter name used in the processing description and the corresponding XML metadata parameter path and tag are listed. Also see the SICD Volume 1 Design & Implementation Description Document, Tables 3-1 through 3-16. In the processing description, scalar quantities such as time, t, are presented in normal face type. Vector quantities, such an the Scene Center Point (SCP) position in ECF coordinates, **SCP**, are listed in bold face type.

Table 1-1 Metadata Parameter List	
Scene Center Point Parameters	
Parameter	**Description**
SCP	SCP position in ECF coordinates (m). XML: SICD.Geodata.SCP.ECF
SCP.Lat, SCP.Lon, SCP.HAE	SCP position in geodetic coordinates (deg, deg, m). May also be computed from SCP position in ECF coordinates. XML: SICD.Geodata.SCP.LLH
SCP_Row, SCP_Col	SCP image pixel grid global row and column indices. XML: SICD.ImageData.SCPPixel
t_{COA}^{SCP}	SCP pixel Center Of Aperture time (sec). Time t = 0 at Collection Start. XML: SICD.SCPCOA.SCPTime
$\mathbf{ARP}_{COA}^{SCP}$	ARP position at the SCP COA time in ECF coordinates (m). XML: SICD.SCPCOA.ARPPos
$\mathbf{VARP}_{COA}^{SCP}$	ARP velocity at the SCP COA time in ECF coordinates (m/sec). XML: SICD.SCPCOA.ARPVel
SideOfTrack	Side of Track for imaging collection (L or R). XML: SICD.SCPCOA.SideOfTrack
$GRAZ_{COA}^{SCP}$	Grazing angle to SCP at SCP COA time (deg). XML: SICD.SCPCOA.GrazAng
Image Data Parameters	
Parameter	**Description**
NumRows, NumCols	Dimensions of the Image pixel array, S_{II}(row,col). The discrete elements of the pixel array are indexed (row,col) XML: SICD.ImageData.NumRows & SICD.ImageData.NumCols
FirstRow, FirstCol	Row index of the first row and column index of the first column of the pixel array. For the full image array, FirstRow = 0 and FirstCol = 0. XML: SICD.ImageData.FirstRow & SICD.ImageData.FirstCol
Image Grid Parameters	
Parameter	**Description**

Table 1-1 Metadata Parameter List	
Grid_Type	Image grid type of the image. Allowed values: RGAZIM, RGZERO, XRGYCR, XATYCT, PLANE XML: SICD.Grid.Type
uRow	Unit vector in the increasing row direction in ECF coordinates. Image plane is spanned by vectors **uRow** & **uCol**. XML: SICD.Grid.Row.UVectECF
uCol	Unit vector in the increasing column direction in ECF coordinates. Image plane is spanned by vectors **uRow** & **uCol**. XML: SICD.Grid.Col.UVectECF
Row_SS	Sample spacing in the row direction (m). XML: SICD.Grid.Row.SS
Col_SS	Sample spacing in the column direction (m). XML: SICD.Grid.Col.SS
cT_COA(m,n), M_TCOA, N_TCOA	Center Of Aperture time polynomial coefficients (sec, sec/m, etc). The polynomial yields COA time (t_{COA}) as a function of image coordinates (xrow, ycol). Time t = 0 at Collection Start. M_TCOA, N_TCOA = Orders of the polynomial in xrow and ycol. XML: SICD.Grid.TimeCOAPoly
ARP Position Parameters	
Parameter	**Description**
cARPx(n), cARPy(n), cARPz(n), N_ARP	Aperture Reference Point polynomial coefficients (m, m/sec, etc.). The polynomials yield ARP position in ECF coordinates as a function of time. Time t = 0 at Collection Start. N_ARP = Order of the polynomials. XML: SICD.Position.ARPPoly
Image Formation Parameters	
Parameter	**Description**
IFA	Image Formation Algorithm (IFA) used to form the image. Allowed values: RGAZCOMP, PFA, RMA, OTHER XML: SICD.ImageFormation.ImageFormAlgo
Range & Azimuth Compression Parameters **Parameters included if and only if IFA = RGAZCOMP**	
Parameter	**Description**
AzSF	Scale factor that scales image coordinate az = ycol (m) to a delta cosine of the Doppler Cone Angle at COA (1/m). $\Delta\cos DCA_{COA}$ = AzSF x az XML: SICD.RgAZComp.AzSF
Polar Format Algorithm Parameters **Parameters included if and only if IFA = PFA**	
Parameter	**Description**

Table 1-1 Metadata Parameter List	
cPA(n), N_PA	Polar Angle polynomial coefficients (rad, rad/sec, etc). The polynomial yields polar angle θ (rad) from collection time. Collection time in seconds (t = 0 at Collection Start). N_PA = Order of polynomial. XML: SICD.PFA.PolarAngPoly
cKSF(n), N_KSF	Spatial Frequency Scale Factor polynomial coefficients (dimensionless, rad^{-1}, rad^{-2}, etc.). The polynomial yields the scale factor KSF from polar angle θ (rad). The scale factor is used to map radio frequency (fx, in Hz) to aperture spatial frequency (Kap, in cycles/m). Kap = fx x 2/c x KSF N_KSF = Order of polynomial. XML: SICD.PFA.SpatialFreqSFPoly
Range Migration Algorithm INCA Parameters **Parameters included if and only if Grid_Type = RGZERO & IFA = RMA.**	
Parameter	**Description**
cT_CA(n), N_TCA	Time of Closest Approach polynomial coefficients (sec, sec/m, etc) for Grid_Type = RGZERO. The polynomial yields time of closest approach (t_CA) as a function of image column coordinate (ycol). Time t = 0 at Collection Start. N_TCA = Order of polynomial. XML: SICD.RMA.INCA.TimeCAPoly
R_{CA}^{SCP}	Range to SCP at closest approach (m). XML: SICD.RMA.INCA.R_CA_SCP
cDRSF(m,n), M_DRSF, N_DRSF	Doppler Rate Scale Factor (DRSF) polynomial coefficients (dimensionless, m^{-1}, m^{-2}, etc). The polynomial yields DRSF as a function of image coordinates (xrow,ycol). M_DRSF, N_DRSF = Orders of the polynomial in xrow and ycol. XML: SICD.RMA.INCA.DRateSFPoly

For any image product, an optional set of parameters may be included that allows the expected accuracy of the image to scene and scene to image projections to be computed. The optional metadata parameters are provided in the Error Statistics parameter block. The Error Statistics parameter block is described in Table 3-12 of the SICD Volume 1 Design & Implementation Description Document. The Error Statistics parameter block allows two options for the set of parameters to be provided. Option 1 is a simple set of estimated statistics for the combined or composite effect of the all sources of projection error. Option 2 is an expanded set of estimated statistics for the various components of image projection error for a SAR system.

The parameters listed in Table 1-2 are derived/computed from the parameters included in a given instance of the Error Statistics block. The covariance matrices and variance parameters are computed based on the common definitions for variance, covariance, standard deviation and correlation coefficient. For each parameter to be computed, the input parameter(s) used from the Error Statistics block are identified by XML parameter path and tag. Also, guidance is provided as needed when optional parameters within the block have been

omitted. For a product that includes both composite and component parameter sets (such a product is permitted but not expected), the component parameters should be used.

The error propagation computations in Section 12 address both composite error statistics and component error statistics. For component error propagation, the computations are defined for the component error sources that (1) are the primary sources of image projection error for a SAR sensor and (2) are correctly applied for any SAR sensor and image product.

Table 1-2 Optional Error Statistics Parameters	
Estimated error statistics that may or may not be included any given product.	
Option 1: Estimated Composite Error Statistics	
Parameter	**Description**
$\mathbf{C}_{\mathrm{RGAZ}}^{\mathrm{SCP}}$	Covariance matrix formed for the composite image error, Δ**RGAZ**, at the SCP. See Section 12. Composite Image Error: $\Delta\mathbf{RGAZ} = \begin{bmatrix} \Delta\mathrm{RG} \\ \Delta\mathrm{AZ} \end{bmatrix}$ Units: meters Matrix size: 2 x 2 Matrix Units: m^2 Matrix elements computed from the product metadata. XML: SICD.ErrorStatistics.CompositeSCP
Option 2: Estimated Component Error Statistics	
Parameter	**Description**
$\mathbf{C}_{\mathrm{ARP_PV}}^{\mathrm{INPUT}}$, FRAME	Covariance matrix for the ARP position & velocity errors, Δ**ARP** and Δ**VARP**, measured SCP COA time. See Section 12. Position Error: $\Delta\mathbf{ARP}_{\mathrm{COA}}^{\mathrm{SCP}} = \begin{bmatrix} \Delta\mathrm{ARP_1} \\ \Delta\mathrm{ARP_2} \\ \Delta\mathrm{ARP_3} \end{bmatrix}$ Units: meters Velocity Error $\Delta\mathbf{VARP} = \begin{bmatrix} \Delta\mathrm{VARP_1} \\ \Delta\mathrm{VARP_2} \\ \Delta\mathrm{VARP_3} \end{bmatrix}$ Units: meters/sec ARP_PV Error: $\Delta\mathbf{ARP_PV} = \begin{bmatrix} \Delta\mathbf{ARP} \\ \Delta\mathbf{VARP} \end{bmatrix}$ Size 6 x 1 FRAME – Reference frame used: ECF, RICF or RICI. See Section 3.2 Matrix Size: 6 x 6 Matrix Units: m^2, m^2/sec and m^2/sec^2 Matrix elements computed from the product metadata. If optional correlation coefficients are omitted, then set all cross correlations to zero. XML: SICD.ErrorStatistics.Components.PosVelErr

Table 1-2 Optional Error Statistics Parameters	
VAR_{RB}	Variance of the radar sensor range bias, ΔR_B. Range bias error is constant across the imaged scene. Range Bias Error: ΔR_B Units: meters Variance Units: m^2 Variance computed from the product metadata standard deviation. XML: SICD.ErrorStatistics.Components.RadarSensor.RangeBias
VAR_{CLK_SF}	Variance of the clock frequency scale factor, CLK_SF, for the radar timing clock that controls both "fast time" and "slow time". Range & Range Rate errors, ΔR_CLK and ΔRDOT_CLK, are proportional to range and range rate (R_{COA} &. $Rdot_{COA}$). Clock Frequency Error: $\Delta f_CLK = CLK_SF \bullet f_CLK_0$ Units: Hz CLK SF Range Error: $\Delta R_CLK = CLK_SF \bullet R_{COA}$ Units: meters CLK SF Rdot Error: $\Delta RDOT_CLK = CLK_SF \bullet Rdot_{COA}$ Units: m/sec Variance Units: dimensionless – CLK_SF is dimensionless Variance computed from the product metadata standard deviation. If omitted, then set variance to zero. XML: SICD.ErrorStatistics.Components.RadarSensor.ClockFreqSF
VAR_{RTV}	Variance of the error in tropospheric delay. Error expressed as a range error for normal incidence, ΔR_TV. Tropo Range Error - Vertical: ΔR_TV Units: meters Error in estimated slant range at COA: $\Delta R_TROP = \Delta R_TV \bullet \frac{1}{\sin(GRAZ_{COA})}$ Variance Units: m^2 Variance computed from the product metadata standard deviation. If omitted, then set variance to zero. XML: SICD.ErrorStatistics.Components.RadarSensor.TropoError
VAR_{RTS}^{SCP}	Variance of the error in troposphere delay. Error expressed as a range error to SCP at COA time, ΔR_TS. Assumed constant across the scene. Tropo Range Error: ΔR_TROP Units: meters Variance Units: m^2 Variance computed from the product metadata standard deviation. If omitted, then set variance to zero. XML: SICD.ErrorStatistics.Components.RadarSensor.TropoError
VAR_{RIV}	Variance of the error in ionosphere delay. Error expressed as a range error for normal incidence, ΔR_IV. Iono Range Error - Vertical: ΔR_IV Units: meters Error in estimated slant range at COA: $\Delta R_IONO = \Delta R_IV \bullet \frac{1}{\sin(GRAZ_{COA})}$ Variance Units: m^2 Variance computed from the product metadata standard deviation. If omitted, then set variance to zero. XML: SICD.ErrorStatistics.Components.RadarSensor.TropoError

1.3 Adjustable Parameters

The error in the geolocation estimates derived from a SAR image product are primarily from two sources. The first is an error in the measured platform position versus time during the imaging collection. The second is a bias in the measured ranges derived from the time of arrival of the received echoes. For a given image product, the accuracy of the geolocation estimates derived from the product can be improved if an estimate of the error in position versus time and/or an estimate of the range bias is available.

The Image To Scene and Scene To Image projections described in the following sections are defined using only the metadata contained in the image product. However, if estimates of the error in measured platform position versus time or the bias in the measured ranges are available, it is straight forward to incorporate the corrections into the projection computations. See Figure 1-3. In the projection computations, the Aperture Reference Point (ARP) position versus time, denoted **ARP**(t), and the range at Center of Aperture, denoted R_{COA}, may be adjusted to correct for the errors. The corrections to be added are referred to as Adjustable Parameter Offsets. The use of the Adjustable Parameter Offsets is described in Section 8.

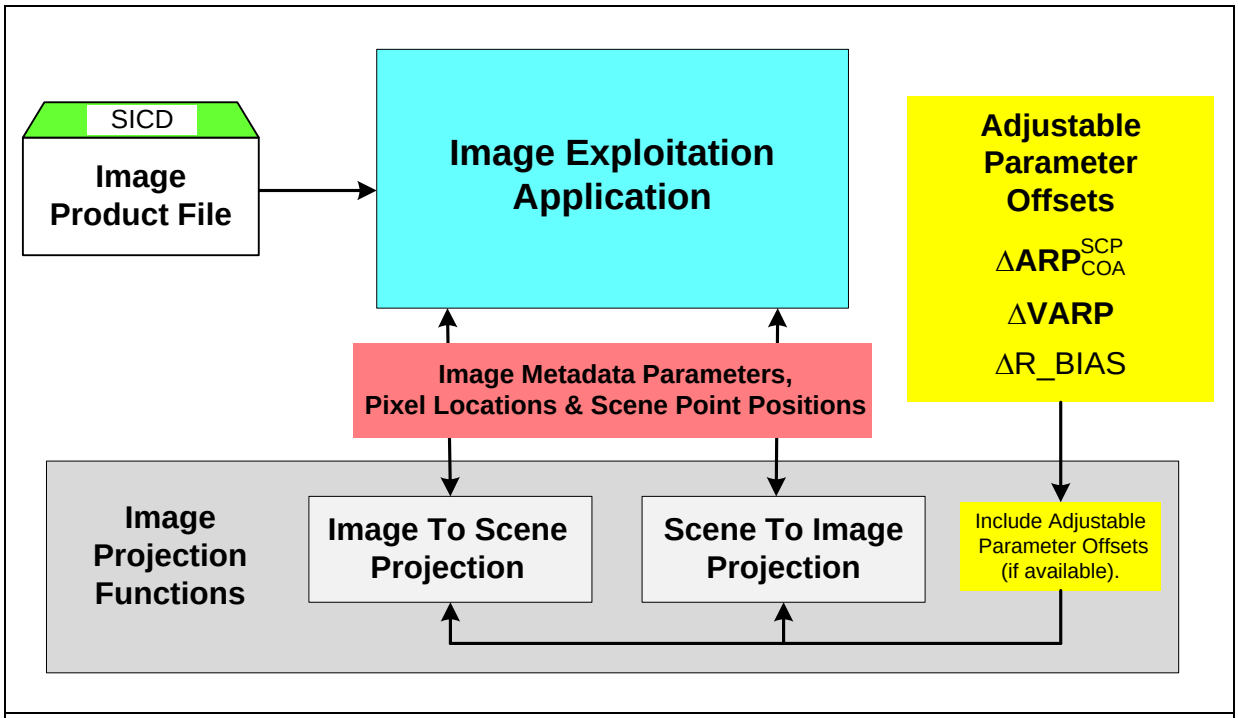


Figure 1-3 Adjustable Parameter Offsets
The Adjustable Parameter Offsets, if available, may be used in all projection computations.

The Adjustable Parameter Offsets are listed in Table 1-3. The ARP position versus time offset, denoted Δ**ARP**(t), is specified by a position offset and velocity offset. The Δ**ARP**(t) offset is the adjustment that is added to the **ARP**(t) included in the image metadata. Ideally, the adjusted **ARP**(t) is the true position versus time for the platform.

$$\underbrace{\text{Adjusted } \mathbf{ARP}(t)}_{\substack{\text{Ideally Equal To The}\\ \text{True Position At Time t}}} = \underbrace{\mathbf{ARP}(t)}_{\substack{\text{Computed from the}\\ \text{Image Metadata}}} + \underbrace{\Delta\mathbf{ARP}_{COA}^{SCP} + \Delta\mathbf{VARP}\bullet\left(t - t_{COA}^{SCP}\right)}_{\substack{\Delta\mathbf{ARP}(t) = \text{ARP Offset}\\ \text{Added To Correct The Error}}}$$

The range bias offset, denoted ΔR_BIAS, is the adjustment that is added to all values of R_{COA} computed from the image metadata. Ideally, for a given pixel location in the image, the adjusted range at COA is the true range to the scene point with impulse response centered at selected pixel location.

$$\underbrace{\text{Adjusted } R_{COA}}_{\substack{\text{Ideally Equal To The}\\ \text{True Range At COA}}} = \underbrace{R_{COA}}_{\substack{\text{Computed from the}\\ \text{Image Metadata}}} + \underbrace{\Delta R_BIAS}_{\substack{\text{Range Bias Offset}\\ \text{Added To Correct The Error}}}$$

The position and velocity offsets defined in Table 1-3 are in ECF coordinates as are the ARP trajectory metadata parameters included with the image products. Applications that estimate platform errors often work in platform-centered coordinates. A commonly used coordinate frame is a Radial, In-Track, Cross-Track (RIC) frame. See Section 3.2 for a description of the RIC coordinate frames. Error estimates initially computed in a platform-centered coordinates are transformed to the ECF frame and then incorporated into the projections.

Note: The method for determining the Adjustable Parameter Offsets is not addressed in this document. For some image products, control points with known geolocations in the imaged scene may be used (if available). For some SAR systems, an improved estimate of the position versus time may become available after the image formation processing is complete. The improved position estimate may be used to compute the ARP offset parameters.

Table 1-3 Adjustable Parameter Offsets	
ARP Offset: $\Delta\mathbf{ARP}(t) = \Delta\mathbf{ARP}_{COA}^{SCP} + \Delta\mathbf{VARP}\bullet\left(t - t_{COA}^{SCP}\right)$	
Parameter	**Description**
$\Delta\mathbf{ARP}_{COA}^{SCP} = \begin{bmatrix} \Delta ARPx_{COA}^{SCP} \\ \Delta ARPy_{COA}^{SCP} \\ \Delta ARPz_{COA}^{SCP} \end{bmatrix}$	ARP Position offset in ECF coordinates (m) that is added to the ARP position at $t = t_{COA}^{SCP}$.
$\Delta\mathbf{VARP} = \begin{bmatrix} \Delta VARPx \\ \Delta VARPy \\ \Delta VARPz \end{bmatrix}$	ARP Velocity offset in ECF coordinates (m/sec) that is added to the $\mathbf{VARP}_{COA}$.
ΔR_BIAS	Range bias offset that is added to the range at COA computed from the image metadata (m).

1.4 Image Projection Computations

The image projections computations described in the following sections are summarized below.

Section 2 Image Pixel Array & Image Plane Parameters

Describes image pixel array and image indexing conventions. Also describes the basis vectors that define the image plane.

Section 3 SCP Pixel Projection

Describes the R/Rdot projection contour for the SCP pixel. The projection contour for the SCP pixel is the same for all image products.

Section 4 Image Grid To R/Rdot Contour

Describes the precise image grid location (xrow,ycol) to R/Rdot contour computation. The computation is a function of the Grid_Type and, in some cases, the Image Formation Algorithm used.

4.1 Image Grid To R/Rdot: Grid_Type = RGAZIM & IFA = PFA

4.2 Image Grid To R/Rdot: Grid_Type = RGAZIM & IFA = RGAZCOMP

4.3 Image Grid To R/Rdot: Grid_Type = RGZERO

4.4 Image Grid To R/Rdot: Grid_Type = XRGYCR

4.5 Image Grid To R/Rdot: Grid_Type = XCTYAT

4.6 Image Grid To R/Rdot: Grid_Type = PLANE

Section 5 Precise R/Rdot To Ground Plane Projection

Describes the precise projection to a ground plane along an R/Rdot contour.

Section 6 Scene Point To Image Grid Projection

Describes the precise scene point to image grid projection using an iterative algorithm.

Section 7 Simple Ground Plane Projection

Describes the simple ground plane projection along straight lines from the image plane. Allows for fast ground plane resampling for any image product.

Section 8 Adjustable Parameters

Describes the use of Adjustable Parameters in the Image To Scene and Scene To Image projections.

Section 9 Precise R/Rdot To Constant HAE Surface Projection

Describes the precise projection to a surface of constant height above the WGS-84 reference ellipsoid along an R/Rdot contour.

Section 10 Precise R/Rdot To DEM Surface Projection

Describes the precise projection to a DEM surface model along an R/Rdot contour.

Section 11 Projection Sensitivity Parameters

Describes the Image to Scene and Scene to Image projection sensitivities for a given image location/scene point projection pair.

Section 12 Projection Error Propagation

Describes the error propagation for both Image To Scene and Scene To Image projections. Projection error statistics are computed based upon the error statistics provided in the image metadata.

2 Image Pixel Array & Image Plane Parameters

The image pixel array is a single two-dimensional array of image signal values. The size of the pixel array is NumRows x NumCols. The elements of the signal array are denoted $\{S_{II}(row,col)\}$. All image products include a Scene Center Point (SCP) position and a SCP pixel location. The spatial sampling represented by the image pixel grid is referred to as the grid type. The image grid type is a function of the imaging collection geometry and the image formation processing that formed the image. Precise image to scene and scene to image projections dependent upon the image grid type.

Image Pixel Array Parameters:

NumRows	Number of rows in the image pixel array.
NumCols	Number of columns in the image pixel array.
FirstRow	Index of the first row of the image pixel array.
FirstCol	Index of the first column of the image pixel array.
Row_SS	Sample spacing in the row direction.
Col_SS	Sample spacing in the column direction.

The image array may be the original image produced by the image formation processor (referred as the full image) or a sub-image extracted from the original image. For the full image array, index FirstRow = 0 and index FirstCol = 0.

2.1 SCP Position & Pixel

The Scene Center Point (SCP) position and image pixel grid location are specified in every image product. The SCP position and pixel location are the base connection point between the image pixel grid and the geolocated three-dimensional scene that was imaged. The SCP row and column indices, SCP_Row and SCP_Col, take on only integer values.

SCP Parameters:

SCP	SCP position in ECF coordinates.
SCP.Lat	SCP geodetic latitude.
SCP.Lon	SCP geodetic longitude.
SCP_Row	SCP image pixel grid global row index.
SCP_Col	SCP image pixel grid global column index.

The image formation processing places the image of the Scene Center Point in the SCP pixel. The SCP pixel is located near the center of the Full Image. For error-free collection and processing, the image impulse response of a point scatterer located at the SCP is centered on the SCP pixel. See Figure 2-1.

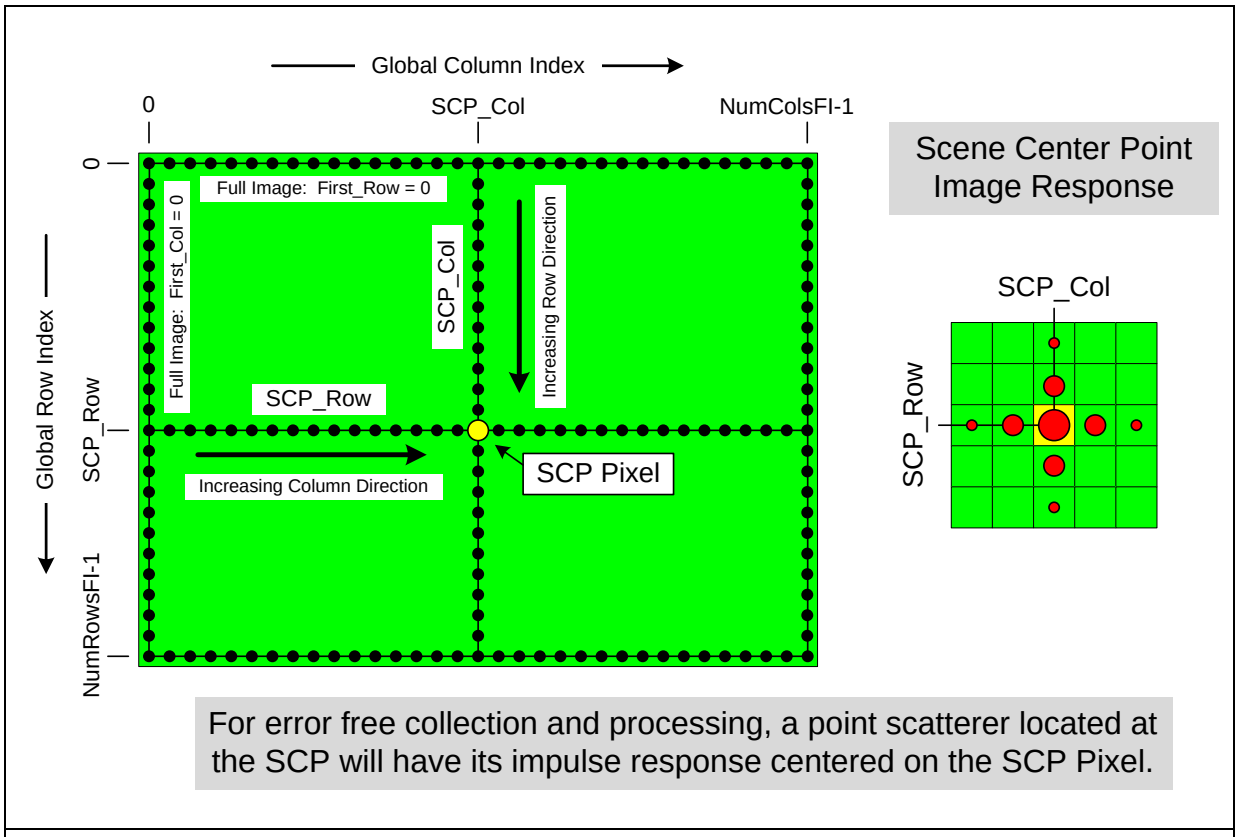


Figure 2-1 SICD Image Pixel Array
The SCP pixel is located near the center of the full image array.

2.2 Image Indices & Image Coordinates

The image grid locations are specified using two sets of pixel based indices: (row, col) and (irow, icol). The image array locations are also specified using distance based image coordinates (xrow, ycol). Indices (row, col) take on only integer values. Indices (irow, icol) and image coordinates (xrow, ycol) take on continuous values. The image sample spacing in the row dimension is Row_SS. The image sample spacing in the column dimension is Col_SS. The row and column spacings are the precise spacings measured at the SCP pixel.

(1) Global Row, Column Indices: (row, col)

The global row, column indices are used to index the pixels of the image array. The origin (0, 0) is located at the first row (row = 0) and the first column of the first row (col = 0) of the Full Image array. In all descriptions, the (row, col) indices are only used to index individual elements of the pixel array $\{S_{II}(\text{row, col})\}$. The global row, column indices take on INTEGER values only. The elements of the full image pixel array are indexed as follows.

Global Row Index: row = 0, 1, 2, . . . , NumRowsFI -1

Global Column Index: col = 0, 1, 2, . . . , NumColsFI -1

(2) SCP Pixel-Centered Image Indices: (irow, icol)

The SCP pixel-centered image indices are continuous valued indices that are also used to index locations in the image pixel array. The origin (0, 0) is located at the CENTER of the SCP pixel. The image indices (irow, icol) are related to global row, column indices (row, col) as follows. See Figure 2-2.

SCP Pixel-Centered Row Index: irow = row - SCP_Row

SCP Pixel-Centered Column Index: icol = col - SCP_Col

In the descriptions that follow, SCP pixel-centered indices (irow, icol) are used to address the image array as a continuous two-dimensional space. The image pixel values $\{S_{II}(row, col)\}$ are considered to be the discrete samples of a continuous, two-dimensional image signal at the integer values of indices (irow, icol).

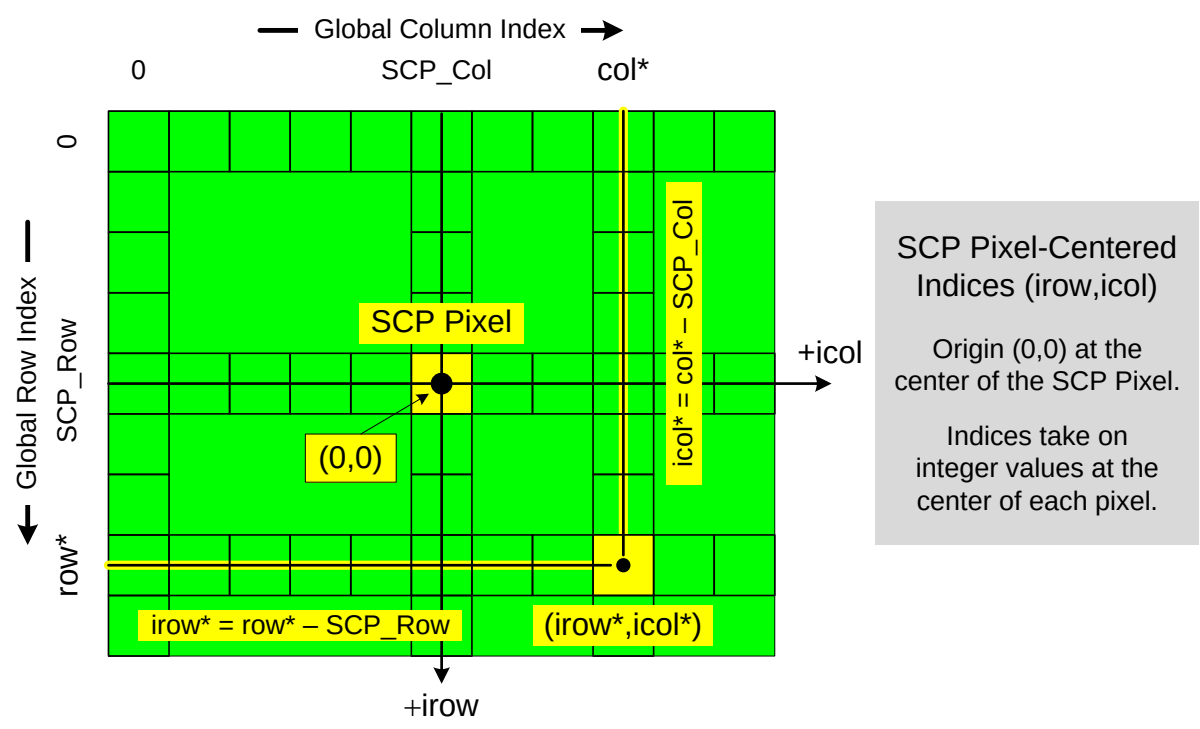


Figure 2-2 SCP Pixel-Centered Indices

Image indices (irow, icol) have origin at the center of the SCP Pixel.

(3) SCP Centered Image Coordinates: (xrow, ycol)

The SCP centered image coordinates are continuous valued distances that treat the image array as equally spaced samples on an orthogonal grid. The origin (0, 0) is located at the CENTER of the SCP pixel. Image coordinate xrow is equal to index irow scaled by the row spacing (Row_SS). Image coordinate ycol is equal to index icol scaled by the column spacing (Col_SS).

$$xrow = Row_SS \bullet irow \qquad\qquad ycol = Col_SS \bullet icol$$

(1) In all SICD descriptions, indices (row, col) are only used to index the discrete elements of the pixel array. Indices (row, col) take on INTEGER values only.

(2) The SCP Pixel-Centered indices (irow,icol) are used to reference the pixel array as a continuous 2-dimensional space. An index value with zero fractional part references the center of the pixel.

(3) Image indices (irow,icol) and image coordinates (xrow,ycol) apply to all SICD images and are used with any grid type.

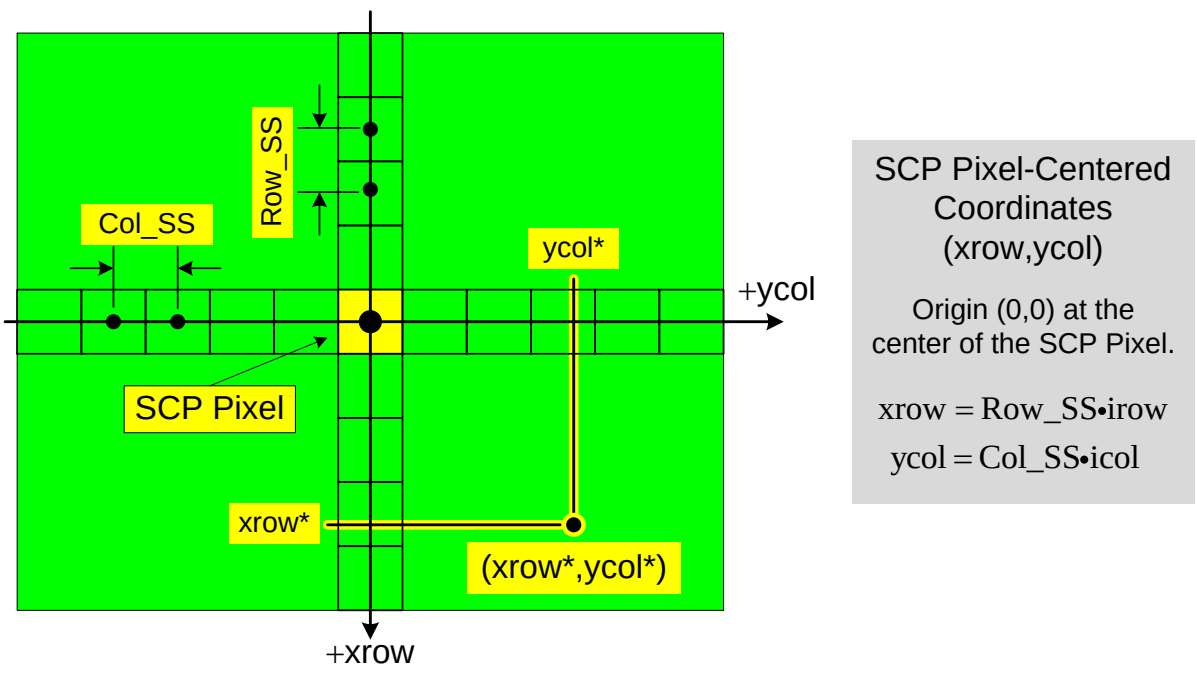


Figure 2-3 SCP Pixel-Centered Coordinates
Image coordinates (xrow,ycol) computed from indices (irow,icol).

2.3 Image Grid Types

The type of image sample grid is a fundamental attribute of the complex image. The spatial sampling of the complex image is usually oriented along the natural radar coordinates range and cross range. The exact grid is a function of both the collection mode and the image formation algorithm. The parameters that identify the grid type and image formation algorithm are as follows.

Image Grid Parameters:

Grid_Type	Identifies the type of spatial sampling of the image grid.
	Allowed values: RGAZIM, RGZERO, XRGYC, XCTYAT, PLANE
IFA	Image Formation Algorithm used to form the image.
	Allowed values: PFA, RMA, RGAZCOMP, OTHER

For a given image grid, the conversion from image grid location to R/Rdot contour is a function of the grid type. For the RGAZIM grid type, it is also dependent upon the image formation algorithm that produced the grid.

(1) Grid_Type = RGAZIM ⇔ Range & Azimuth

Samples are oriented in range and azimuth with respect to the ARP position and velocity at an image formation reference time (denoted $\mathbf{ARP}_{\mathrm{REF}}$ and $\mathbf{VARP}_{\mathrm{REF}}$). Image rows are samples along nominally constant range contours relative to the $\mathbf{ARP}_{\mathrm{REF}}$. Image columns are nominally oriented along constant azimuth. Along a given row, the image grid samples are evenly spaced increments in an azimuth angle related quantity (usually the cosine of the Doppler cone angle). The RGAZIM grid is the natural output for image formation algorithms RGAZCOMP and PFA. The RGAZCOMP algorithm (simple range and azimuth compression) is only used for coarse resolution or small scene sizes. The PFA algorithm was developed to overcome the limitations of RGAZCOMP to yield fine resolution and large scene sizes.

(2) Grid_Type = RGZERO ⇔ Range & Zero Doppler

The Range and Zero Doppler grid is for imaging collections near closest approach (i.e. near zero Doppler frequency). The grid is produced using a variation of the RMA for near broadside imaging. Image rows are samples along contours of constant range from the ARP trajectory and are “parallel” to the ARP trajectory. Image columns are samples along a contour of constant time of closest approach with range increasing in the increasing row direction.

(3) Grid_Type = XRGYCR ⇔ X,Y Grid Oriented Range, Cross Range

The XRGYCR grid is a slant plane grid that is oriented range and cross range. The slant plane is defined by the ARP position and velocity at a reference time ($\mathbf{POS}_{\mathrm{REF}}$ and $\mathbf{VEL}_{\mathrm{REF}}$) and the SCP. The +XRG direction is oriented from the ARP to the SCP (in the increasing

range direction). The +YCR direction is orthogonal to the +XRG direction (in the cross range direction). The rows of the image are along lines of constant XRG and evenly spaced. The columns of the image are along lines of constant YCR and evenly spaced.

(4) Grid_Type = XCTYAT ⇔ X,Y Grid Oriented Cross Track, Along Track

The XCTYAT grid is a slant plane grid that is oriented cross track and along track. The slant plane is defined by the ARP position and velocity at a reference time ($\mathbf{POS}_{REF}$ and $\mathbf{VEL}_{REF}$) and the SCP. The +XCT direction is orthogonal to the $\mathbf{VEL}_{REF}$ and in the increasing distance from the trajectory. The +YAT direction is orthogonal to the +XCT direction and parallel to the $\mathbf{VEL}_{REF}$. For right looking collections, the +YAT direction is in the same direction as $\mathbf{VEL}_{REF}$. For left looking collections, the +YAT is in the opposite direction as $\mathbf{VEL}_{REF}$. The rows of the image are along lines of constant XCT and evenly spaced. The columns of the image are along lines of constant YAT and evenly spaced. The grid is the natural output for image formation using the RMA.

(5) Grid_Type = PLANE ⇔ Arbitrary Plane w/ Axes U,V

The PLANE grid is provided for images that do not meet the precise definition of the other grid types. The samples are uniformly spaced in a geolocated image plane that contains the SCP. The +U coordinate is equal to the row coordinate (xrow). The +V coordinate is equal to the column coordinate (ycol). The rows of the image are along lines of constant U coordinate and are evenly spaced. The columns of the image are along lines of constant V coordinate and are evenly spaced. The only restriction is the image be a view from above the earth.

2.4 Image Plane Parameters

The samples of the image pixel grid may be described as a grid of uniformly spaced samples in a geolocated Image Plane. For image grid types XRGYCR and XCTYAT, this is precisely true. For grid types RGAZIM and RGZERO, this is only approximately true but is still a useful representation of the image sampling. The mapping from image grid location (xrow, ycol) to image plane position is described below.

Image Plane Parameters:

uRow	Unit vector in the increasing row direction in ECF coordinates.
uCol	Unit vector in the increasing column direction in ECF coordinates.

The Image Plane is defined by the SCP position and two unit vectors that lie in the plane. The unit vector in the row direction (**uRow**) represents the change in image plane position moving from the SCP pixel in the increasing row direction. The unit vector in the column direction (**uCol**) represents the change in image plane position moving from the SCP pixel in the increasing column direction.

The Image Plane unit normal vector (**uIPN**) in ECF coordinates is computed as follows. For all image products, the IPN will point away from the center of the earth.

$$\mathbf{IPN} = \mathbf{uRow} \times \mathbf{uCol} \qquad \mathbf{uIPN} = \frac{1}{|\mathbf{IPN}|} \bullet \mathbf{IPN}$$

For image grid location (xrow, ycol), the corresponding Image Plane Point, **IPP**(xrow, ycol), in ECF coordinates is computed as shown below. The displacement from the SCP to the IPP is denoted Δ**IPP**(xrow, ycol). See Figure 2-4.

$$\Delta\mathbf{IPP}(\text{xrow, ycol}) = \text{xrow} \bullet \mathbf{uRow} + \text{ycol} \bullet \mathbf{uCol}$$

$$\mathbf{IPP}(\text{xrow, ycol}) = \mathbf{SCP} + \Delta\mathbf{IPP}(\text{xrow, ycol})$$

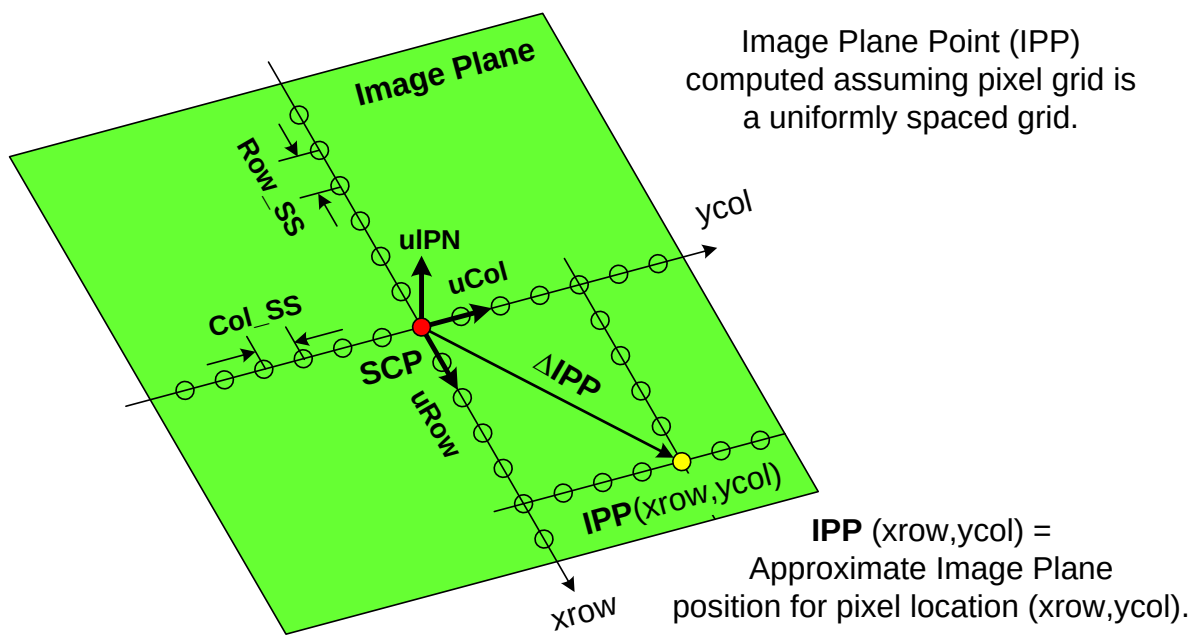


Figure 2-4 Image Plane Parameters
The Image Plane is defined by the SCP and unit vectors uRow & uCol.

For most image products, vectors **uRow** and **uCol** are an orthogonal pair ($\mathbf{uRow} \bullet \mathbf{uCol} = 0$), however, they are not constrained be an orthogonal pair. Define the angle θ_{COL} as the angle between the vectors. See Figure 2-5. Compute the cosine and sine of θ_{COL} as follows.

$$\cos(\theta_{COL}) = \mathbf{uRow} \bullet \mathbf{uCol} \qquad \sin(\theta_{COL}) = +(1 - \cos^2(\theta_{COL}))^{1/2}$$

For a given point in the image plane at position **IPP**, the corresponding SCP pixel-centered coordinates (xrow, ycol) are computed as follows.

$$\Delta\mathbf{IPP} = \mathbf{IPP} - \mathbf{SCP}$$

$$\begin{bmatrix} \text{xrow} \\ \text{ycol} \end{bmatrix} = \frac{1}{\sin^2(\theta_{COL})} \bullet \begin{bmatrix} 1 & -\cos(\theta_{COL}) \\ -\cos(\theta_{COL}) & 1 \end{bmatrix} \bullet \begin{bmatrix} \Delta\mathbf{IPP} \bullet \mathbf{uRow} \\ \Delta\mathbf{IPP} \bullet \mathbf{uCol} \end{bmatrix}$$

Converting from an arbitrary image plane position to image grid location (xrow, ycol) will, in general, yield SCP pixel-centered indices (irow, icol) with fractional values.

$$\text{irow} = \frac{1}{\text{Row_SS}} \bullet \text{xrow} \qquad \text{icol} = \frac{1}{\text{Col_SS}} \bullet \text{ycol}$$

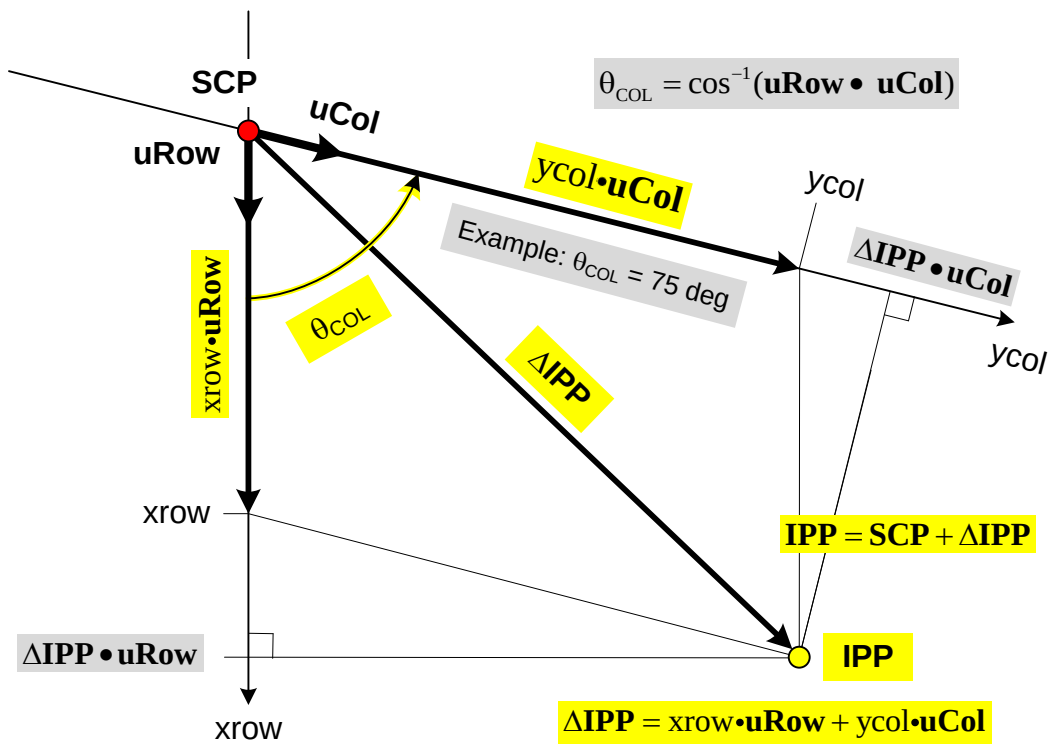


Figure 2-5 Image Plane Vectors
Unit vectors uRow & uCol are allowed to be non-orthogonal.

2.5 Image Grid To COA Parameters

Accurate image projection R/Rdot contours are measured relative to the ARP position and velocity at the Center Of Aperture time (denoted t_{COA}). The COA time is allowed to vary smoothly across the image grid.

Center Of Aperture Parameters:

cT_COA(m,n)	Center Of Aperture time polynomial coefficients. The polynomial yields COA time as a function of image coordinates (xrow, ycol) in meters. COA time in seconds (t = 0 at Collection Start).
M_TCOA	Order of the COA time polynomial in the Row Coordinate.
N_TCOA	Order of the COA time polynomial in the Column Coordinate.

Aperture Reference Point Parameters:

cARPx(n)	Aperture Reference Point position polynomial coefficients. The polynomial yields ARP position ECF X coordinate as a function of time (t = 0 at Collection Start).
cARPy(n)	ARP ECF Y coordinate coefficients.
cARPz(n)	ARP ECF Z coordinate coefficients.
N_ARP	Order of the ARP position polynomials.

The t_{COA} is specified as a two-dimensional polynomial of image coordinates (xrow, ycol). Polynomial coefficients are denoted cT_COA(m,n). The polynomial is of order M_TCOA in the row coordinate and N_TCOA in the column coordinate. The t_{COA} for image grid location (xrow, ycol) is computed as shown below.

$$t_{COA}(\text{xrow, ycol}) = \sum_{m=0}^{\text{M_TCOA}} \sum_{n=0}^{\text{N_TCOA}} \text{cT_COA}(m,n) \bullet (\text{xrow})^m \bullet (\text{ycol})^n$$

The t_{COA}(xrow, ycol) computed above is the precise value of t_{COA} for the pixel location. Even though image coordinates (xrow, ycol) may be approximate displacements, the computed t_{COA} is the time to be used for accurate exploitation. The choice of the orders M_TCOA and N_TCOA are made such that the resulting t_{COA} values accurately match those achieved by the image formation processing.

For all images, coefficient cT_COA(0,0) is the COA time for the SCP pixel (t_{COA}^{SCP}). Images with constant t_{COA} (typically from spotlight collections) may be stored with only the cT_COA(0,0) coefficient and order parameters M_TCOA = N_TCOA = 0. An example image for which the COA time varies across the image is shown in Figure 2-6. Shown in the figure are two contours of constant t_{COA}. For most images, the contours will be approximately linear and equally spaced for even increments in t_{COA}.

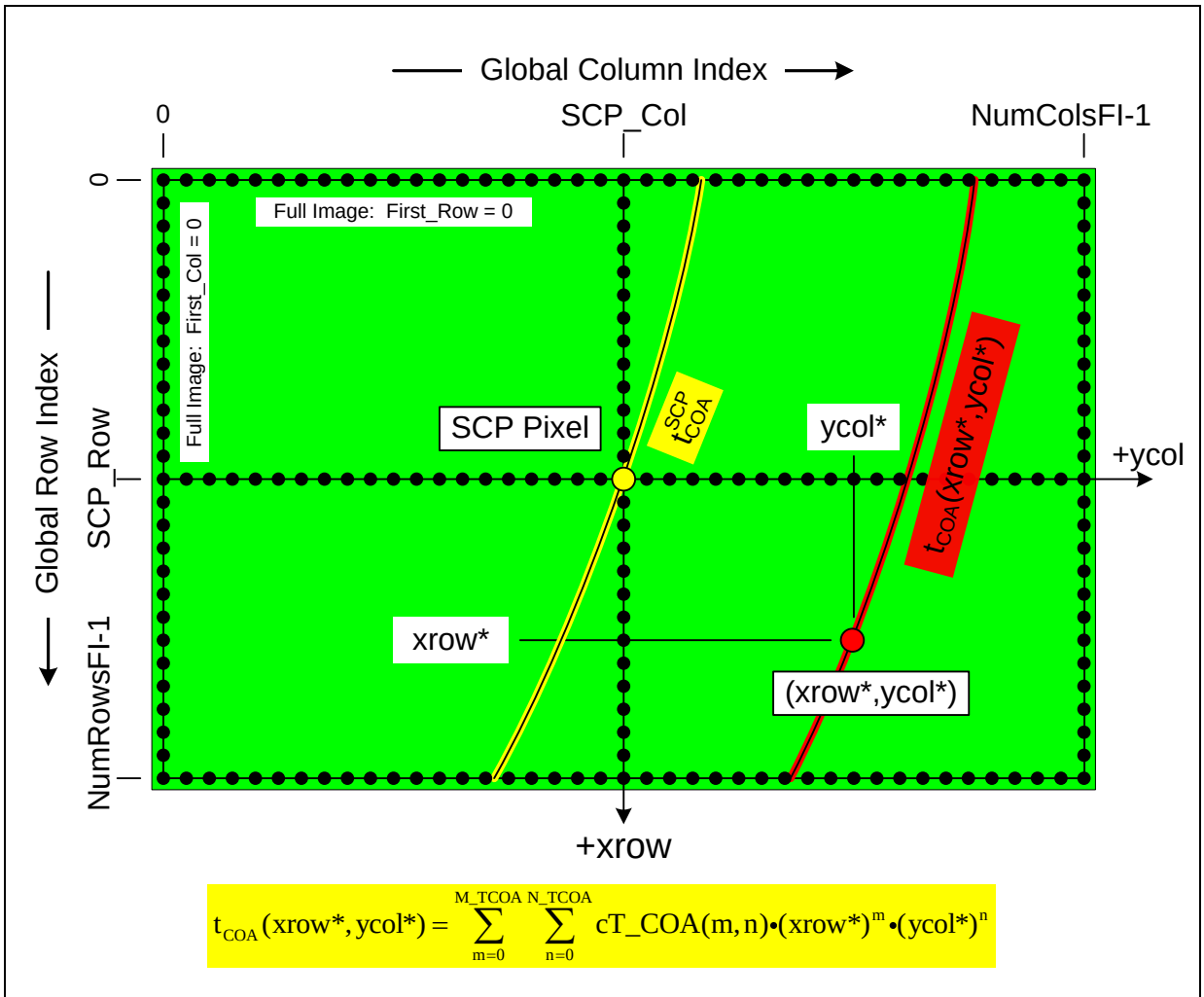


Figure 2-6 Precise Center Of Aperture Time

Example shows contour lines of constant t_{COA} for the SCP pixel and an arbitrary grid location (xrow*, ycol*).

For a given pixel, the R/Rdot projection contour is computed relative to the ARP position and velocity at the pixel COA time. See Figure 2-7. The ARP COA position and velocity are computed as follows. The precise computations of the R_{COA} and $Rdot_{COA}$ are described in Section 4.

$$\mathbf{ARP}_{COA} = \begin{bmatrix} ARPx(t_{COA}) \\ ARPy(t_{COA}) \\ ARPz(t_{COA}) \end{bmatrix} = \begin{bmatrix} \sum_{n=0}^{N_ARP} cARPx(n) \bullet (t_{COA})^n \\ \sum_{n=0}^{N_ARP} cARPy(n) \bullet (t_{COA})^n \\ \sum_{n=0}^{N_ARP} cARPz(n) \bullet (t_{COA})^n \end{bmatrix}$$

$$\mathbf{VARP}_{COA} = \begin{bmatrix} VARPx(t_{COA}) \\ VARPy(t_{COA}) \\ VARPz(t_{COA}) \end{bmatrix} = \begin{bmatrix} \sum_{n=1}^{N_ARP} n \bullet cARPx(n) \bullet (t_{COA})^{n-1} \\ \sum_{n=1}^{N_ARP} n \bullet cARPy(n) \bullet (t_{COA})^{n-1} \\ \sum_{n=1}^{N_ARP} n \bullet cARPz(n) \bullet (t_{COA})^{n-1} \end{bmatrix}$$

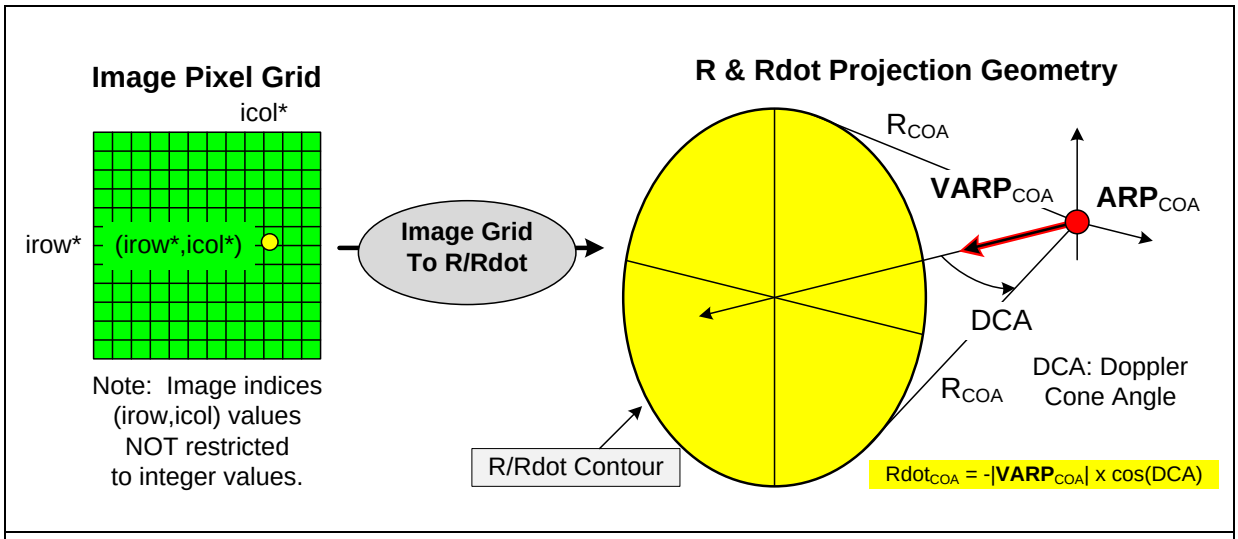


Figure 2-7 R/Rdot Relative To The ARP At COA

Image location (irow*, icol*) projection contour relative to ARP position and velocity at t_{COA}.

3 SCP Pixel Projection

The Scene Center Point (SCP) position and image pixel grid location are specified in every image product. The SCP position and pixel location are the base connection point between the image pixel grid and the georeferenced three-dimension scene that was imaged. The SCP position, image pixel location and Center Of Aperture parameters are provided with all products and are summarized below. All positions and velocities are in ECF coordinates.

SCP Center Of Aperture Parameters:

t_{COA}^{SCP}	SCP pixel Center Of Aperture time.
$\mathbf{ARP}_{COA}^{SCP}$	ARP position at the SCP COA time in ECF coordinates.
$\mathbf{VARP}_{COA}^{SCP}$	ARP velocity at the SCP COA time in ECF coordinates.
SideOfTrack	Side of Track parameter. SideOfTrack = L ⇔ Left Looking SideOfTrack = R ⇔ Right Looking

3.1 SCP Projection Equations

The SCP pixel R/Rdot projection contour is computed from the SCP COA parameters. The SCP range and range rate relative the ARP at COA are computed as follows.

$$R_{COA}^{SCP} = \left|\mathbf{ARP}_{COA}^{SCP} - \mathbf{SCP}\right|$$

$$Rdot_{COA}^{SCP} = \frac{1}{R_{COA}^{SCP}} \bullet \mathbf{VARP}_{COA}^{SCP} \bullet (\mathbf{ARP}_{COA}^{SCP} - \mathbf{SCP})$$

The Side of Track parameter is provided for convenience. It can also be computed from the ARP position versus time. Define parameter LOOK that is used in later computations to select ground points on the correct side of the ground track.

$$LOOK = \begin{cases} +1, & \text{if SideOfTrack = L} \\ -1, & \text{if SideOfTrack = R} \end{cases}$$

The normal to the instantaneous slant plane that contains the SCP at the SCP COA time is computed as follows. See Figure 3-1. The slant plane unit normal ($\mathbf{uSPN}_{COA}^{SCP}$) is tangent to R/Rdot contour at the SCP. The normal vector computed points away from the center of the earth.

$$\mathbf{SPN}_{COA}^{SCP} = LOOK \bullet \mathbf{VARP}_{COA}^{SCP} \times (\mathbf{SCP} - \mathbf{ARP}_{COA}^{SCP}) \qquad \mathbf{uSPN}_{COA}^{SCP} = \frac{1}{\left|\mathbf{SPN}_{COA}^{SCP}\right|} \bullet \mathbf{SPN}_{COA}^{SCP}$$

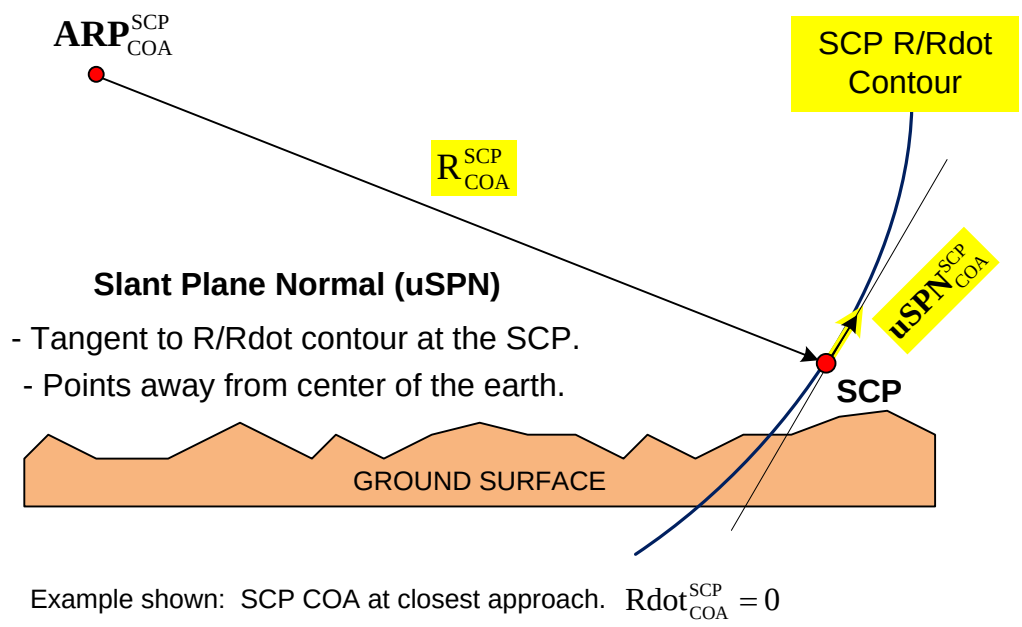


Figure 3-1 SCP Pixel R/Rdot Contour Projection
The SCP pixel R/Rdot contour is computed from the SCP position and ARP & VARP at COA.

3.2 RIC Coordinates At SCP COA

Applications that estimate platform errors and error statistics often work in a platform-centered coordinate frame. A commonly used coordinate frame is a Radial, In-Track, Cross-Track (RIC) frame. The RIC frame is defined at a given time based on the instantaneous position and velocity of the platform. The origin of the RIC frame is the instantaneous position of the platform. The radial direction is from the earth center (i.e. the ECF origin) to the platform pointing away from the earth center. The cross-track direction is normal to the radial direction and the instantaneous velocity. The in-track direction completes a right-handed coordinate frame. For airborne platforms, the ECF velocity vector is commonly used and the resulting RIC frame is referred to as the RICF frame. For orbiting platforms, the ECI velocity vector is most commonly used and the resulting RIC frame is referred to as the RICI frame.

For a given SICD product, the RIC frame at the SCP COA time is used for transforming estimated error statistic estimates from an RIC frame to the ECF frame. For both the RICF and RICI frames, the transformation to ECF coordinates is computed using the ARP position and velocity at t^{SCP}_{COA}. An example RICF frame at time t^{SCP}_{COA} is shown in Figure 3-2.

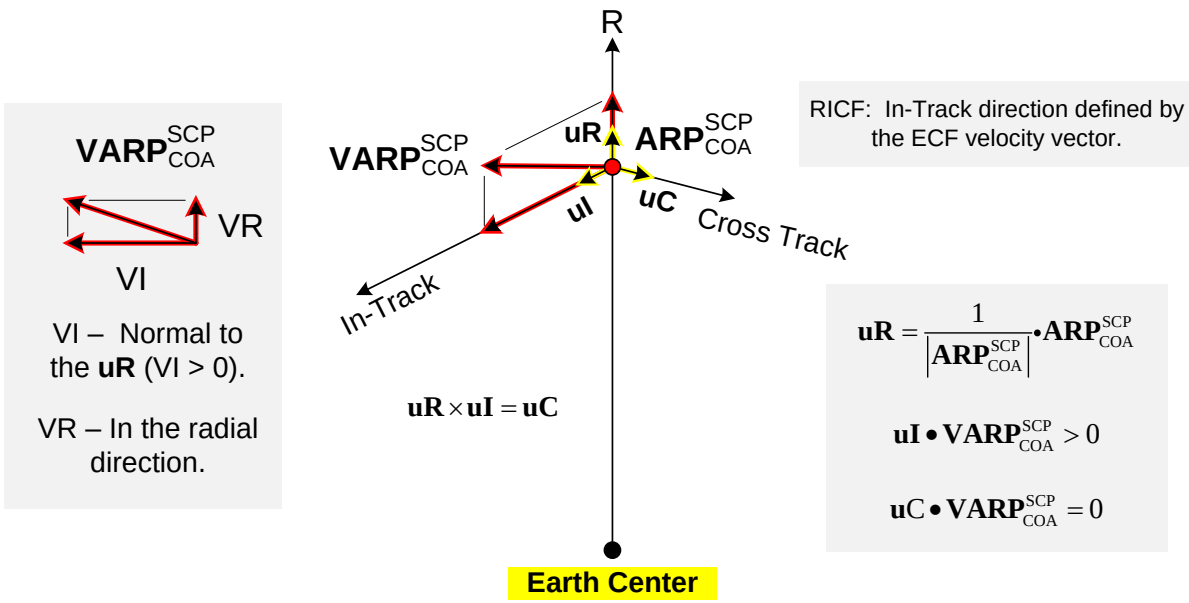


Figure 3-2 RICF Coordinates

The ARP position and ECF velocity vector at SCP COA define the RICF coordinates.

The RICF basis vectors (**uR**, **uI** and **uC)** are computed as follows. The transformation matrix from RICF to ECF coordinates is formed from the basis vectors.

$$\mathbf{uR} = \frac{1}{\left|\mathbf{ARP}_{\mathrm{COA}}^{\mathrm{SCP}}\right|} \bullet \mathbf{ARP}_{\mathrm{COA}}^{\mathrm{SCP}} \qquad \mathbf{uC} = \frac{1}{\left|\mathbf{uR} \times \mathbf{VARP}_{\mathrm{COA}}^{\mathrm{SCP}}\right|} \bullet \left(\mathbf{uR} \times \mathbf{VARP}_{\mathrm{COA}}^{\mathrm{SCP}}\right)$$

$$\mathbf{uI} = \mathbf{uC} \times \mathbf{uR} \qquad \mathrm{T}_{\mathrm{RICF}}^{\mathrm{ECF}} = \underbrace{\begin{bmatrix} | & | & | \\ \mathbf{uR} & \mathbf{uI} & \mathbf{uC} \\ \downarrow & \downarrow & \downarrow \end{bmatrix}}_{\substack{\text{Matrix formed from} \\ \text{the RICF basis vectors.}}}$$

For the RICI frame, the radial direction is the same as the RICF radial direction. The in-track and cross-track directions are computed from the ARP ECI velocity vector. The ECI velocity vector in ECF coordinates, denoted $\mathbf{V_ECI}_{\mathrm{COA}}^{\mathrm{SCP}}$, is computed as follows. Parameter ω_3 is the nominal rotation rate of the ECF frame relative to the ECI frame.

$$\mathbf{V_ECI}_{\mathrm{COA}}^{\mathrm{SCP}} = \mathbf{VARP}_{\mathrm{COA}}^{\mathrm{SCP}} + \begin{pmatrix} 0 \\ 0 \\ \omega_3 \end{pmatrix} \times \mathbf{ARP}_{\mathrm{COA}}^{\mathrm{SCP}} \qquad \omega_3 = 0.000{,}072{,}921 \text{ rad/sec}$$

The RICI basis vectors (**uRI**, **uII** and **uCI)** are computed as follows. The transformation matrix from RICI to ECF coordinates is formed from the basis vectors.

$$\mathbf{uRI} = \frac{1}{\left|\mathbf{ARP}_{\mathrm{COA}}^{\mathrm{SCP}}\right|} \bullet \mathbf{ARP}_{\mathrm{COA}}^{\mathrm{SCP}} \qquad \mathbf{uCI} = \frac{1}{\left|\mathbf{uRI} \times \mathbf{V_ECI}_{\mathrm{COA}}^{\mathrm{SCP}}\right|} \bullet \left(\mathbf{uRI} \times \mathbf{V_ECI}_{\mathrm{COA}}^{\mathrm{SCP}}\right)$$

$$\mathbf{uII} = \mathbf{uCI} \times \mathbf{uR} \qquad \mathrm{T}_{\mathrm{RICI}}^{\mathrm{ECF}} = \underbrace{\begin{bmatrix} | & | & | \\ \mathbf{uRI} & \mathbf{uII} & \mathbf{uCI} \\ \downarrow & \downarrow & \downarrow \end{bmatrix}}_{\text{Matrix formed from the RICI basis vectors.}}$$

Note: For the purposes of computing the ECI velocity vector, the relationship between the ECI coordinate frame and the ECF coordinate frame is approximated by a single rotation about the z-axis. The rotation about the z-axis is $\alpha(t)$ with nominal rotation rate ω_3. The rotation matrix $R_3(t)$ relates the two frames. The skew symmetric matrix Ω_3 is used to compute the derivative of the $R_3(t)$ rotation matrix. Matrix Ω_3 is useful in transforming position and velocity error covariance matrices from the RICI frame to the ECF frame. See Section 12.4.

$$\mathbf{P}_{\mathrm{ECF}}(t) = R_3(t) \bullet \mathbf{P}_{\mathrm{ECI}}(t) \qquad \alpha(t) = \alpha_0 + \omega_3 \bullet (t - t_0)$$

$$R_3(t) = \underbrace{\begin{bmatrix} \cos(\alpha(t)) & \sin(\alpha(t)) & 0 \\ -\sin(\alpha(t)) & \cos(\alpha(t)) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\text{Rotation from ECI to ECF at time t.}} \qquad \Omega_3 = \begin{bmatrix} 0 & \omega_3 & 0 \\ -\omega_3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{d}{dt} R_3(t) = \omega_3 \bullet \begin{bmatrix} -\sin(\alpha(t)) & \cos(\alpha(t)) & 0 \\ -\cos(\alpha(t)) & -\sin(\alpha(t)) & 0 \\ 0 & 0 & 0 \end{bmatrix} = \Omega_3 \bullet R_3(t)$$

4 Image Grid To R/Rdot Contour

The precise image pixel grid location to R/Rdot projection contour computations are described in following subsections. For the image grid location specified by image pixel indices (irow,icol) or image coordinates (xrow,ycol), image COA time (t_{COA}) and COA ARP position and velocity (ARP_{COA} and $VARP_{COA}$) are computed as described in Section 2. The precise computation of the R/Rdot contour is dependent upon the image grid type and the image formation algorithm that produced the image. See Figure 4-1. The computations for the combinations of grid types and IFAs are described in Sections 4.1 through 4.6.

Image Grid Parameters:

Grid_Type	Image grid type represented by the sampling. Allowed values: RGAZIM, RGZERO, XRGYCR, XCTYAT, PLANE.
IFA	Image Formation Algorithm used to produce the grid. Allowed values: PFA, RMA, RGAZCOMP, OTHER

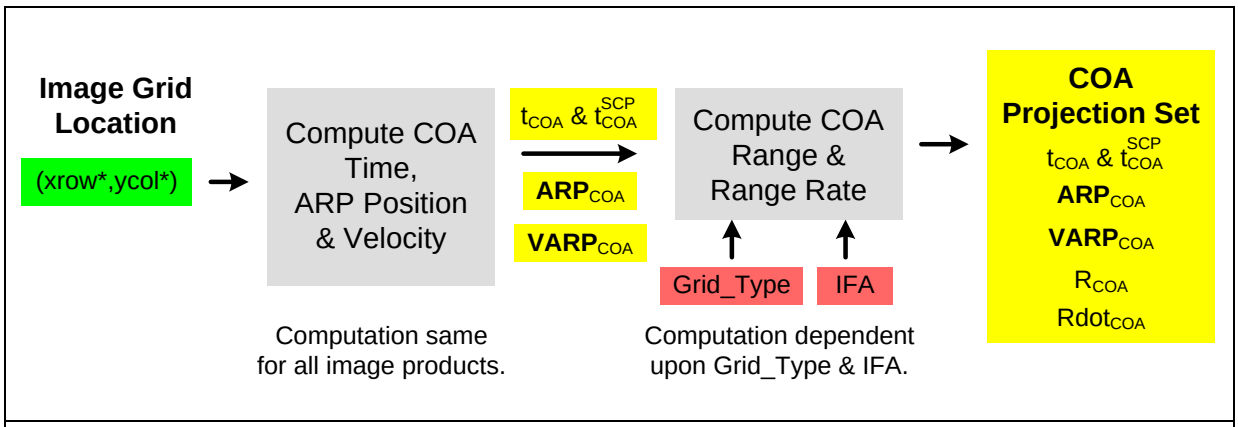


Figure 4-1 Precise R/Rdot Computation
The precise R/Rdot are dependent upon the Grid_Type and, for some grid types, the Image Formation Algorithm (IFA) used to form the image.

The set of parameters that define the R/Rdot projection contour is referred to as the COA Projection Set. The COA Projection Set consists of the 6 parameters. See Figure 4.1.

$$\text{COA Projection Set: } \{\ t_{COA} \quad t_{COA}^{SCP} \quad \mathbf{ARP}_{COA} \quad \mathbf{VARP}_{COA} \quad R_{COA} \quad Rdot_{COA}\ \}$$

The SCP COA time is included for use in computing the error in image projections due to an error in ARP velocity. It is also needed for computing projections that make use of Adjustable Parameter Offsets. The use of Adjustable Parameter Offsets is described in Section 8.

4.1 Image Grid To R/Rdot: Grid_Type = RGAZIM & IFA = PFA

For the RGAZIM grid, the image coordinates are range and azimuth. The row coordinate is the range coordinate, xrow = rg. The column coordinate is the azimuth coordinate, ycol = az. For image pixel location (irow,icol), the (rg,az) coordinates are computed as follows.

$$\mathrm{rg} = \mathrm{xrow} = \mathrm{Row_SS} \bullet \mathrm{irow} \qquad \mathrm{az} = \mathrm{ycol} = \mathrm{Col_SS} \bullet \mathrm{icol}$$

For images formed with the Polar Format Algorithm, the R/Rdot computation makes use of the following SICD PFA specific parameters.

PFA Parameters:

cPA(n)	Polar Angle polynomial coefficients. The polynomial yields polar angle (θ) in radians from collection time. Collection time in seconds (t = 0 at Collection Start).
N_PA	Order of the Polar Angle polynomial.
cKSF(n)	Spatial Frequency Scale Factor polynomial coefficients. The polynomial yields the scale factor (KSF) from polar angle (θ). The scale factor is used to map transmit frequency (fx, in Hz) to aperture spatial frequency (Kap, in cycles/m). $\mathrm{Kap}(\mathrm{fx},\theta) = \mathrm{fx} \bullet \frac{2}{c} \bullet \mathrm{KSF}(\theta)$.
N_KSF	Order of the Spatial Frequency Scale Factor polynomial.

The computation of the R/Rdot contour for an arbitrary image grid location is presented with minimal explanation in order that the necessary computations are clearly presented. Refer to Figures 4-2 and 4-3.

PFA Image Grid Location To R/Rdot Contour

(1) Consider a target (TGT) with impulse response centered at image grid location (irow^{TGT}, icol^{TGT}). Grid location (irow^{TGT}, icol^{TGT}) is converted to image coordinates (rg^{TGT}, az^{TGT}). The COA parameters are computed for image coordinates (rg^{TGT}, az^{TGT}).

$$(\mathrm{irow}^{TGT}, \mathrm{icol}^{TGT}) \;\rightarrow\; (\mathrm{rg}^{TGT}, \mathrm{az}^{TGT}) \;\rightarrow\; t_{COA}^{TGT} \;\rightarrow\; \mathbf{ARP}_{COA}^{TGT} \;\&\; \mathbf{VARP}_{COA}^{TGT}$$

(2) Compute the range and range rate to the SCP at the pixel COA time as follows.

$$(R^{SCP})_{COA}^{TGT} = \left|\mathbf{ARP}_{COA}^{TGT} - \mathbf{SCP}\right| \qquad (\mathrm{Rdot}^{SCP})_{COA}^{TGT} = \frac{1}{(R^{SCP})_{COA}^{TGT}} \bullet \mathbf{VARP}_{COA}^{TGT} \bullet (\mathbf{ARP}_{COA}^{TGT} - \mathbf{SCP})$$

(3) Compute the polar angle (θ) and its derivative with respect to time (dθ/dt) at the pixel COA time as follows.

$$\theta_{COA}^{TGT} = \sum_{n=0}^{N_PA} cPA(n) \bullet (t_{COA}^{TGT})^{n} \qquad (d\theta/dt)_{COA}^{TGT} = \sum_{n=1}^{N_PA} n \bullet cPA(n) \bullet (t_{COA}^{TGT})^{n-1}$$

(4) Compute the polar aperture scale factor (KSF) and its derivative with respect to polar angle (dKSF/dθ) at the pixel COA time as follows.

$$KSF_{COA}^{TGT} = \sum_{n=0}^{N_KSF} cKSF(n) \bullet (\theta_{COA}^{TGT})^{n} \qquad (dKSF/d\theta)_{COA}^{TGT} = \sum_{n=1}^{N_KSF} n \bullet cKSF(n) \bullet (\theta_{COA}^{TGT})^{n-1}$$

(5) Compute the spatial frequency domain phase slopes in the radial (Ka) and cross radial (Kc) directions ($\delta\Phi^{TGT}/\delta Ka$ and $\delta\Phi^{TGT}/\delta Kc$) for the radial direction at θ_{COA}^{TGT}. Note: The sign parameter (SGN) for the phase may be ignored as it is cancelled in a subsequent computation.

$$(\delta\Phi^{TGT}/\delta Ka)_{COA}^{TGT} = rg^{TGT} \bullet \cos(\theta_{COA}^{TGT}) + az^{TGT} \bullet \sin(\theta_{COA}^{TGT})$$

$$(\delta\Phi^{TGT}/\delta Kc)_{COA}^{TGT} = -rg^{TGT} \bullet \sin(\theta_{COA}^{TGT}) + az^{TGT} \bullet \cos(\theta_{COA}^{TGT})$$

(6) Compute range relative to the SCP (ΔR^{TGT}) at the COA.

$$\Delta R_{COA}^{TGT} = KSF_{COA}^{TGT} \bullet (\delta\Phi^{TGT}/\delta Ka)_{COA}^{TGT}$$

(7) Compute the derivative of the range relative to the SCP with respect to polar angle ($d(\Delta R^{TGT})/d\theta$) at the COA. Scale by the derivative of the polar angle with respect to time to yield the derivative with respect to time ($d(\Delta R^{TGT})/dt = \Delta Rdot^{TGT}$).

$$\left(d(\Delta R^{TGT})/d\theta\right)_{COA}^{TGT} = (dKSF/d\theta)_{COA}^{TGT} \bullet (\delta\Phi^{TGT}/\delta Ka)_{COA}^{TGT} + KSF_{COA}^{TGT} \bullet (\delta\Phi^{TGT}/\delta Kc)_{COA}^{TGT}$$

$$\Delta Rdot_{COA}^{TGT} = \left(d(\Delta R^{TGT})/d\theta\right)_{COA}^{TGT} \bullet (d\theta/dt)_{COA}^{TGT}$$

(8) Compute the range and range rate relative to the ARP at COA (R_{COA}^{TGT} and $Rdot_{COA}^{TGT}$). The projection to three-dimensional scene point for grid location (rg^{TGT}, az^{TGT}) is along this R/Rdot contour.

$$R_{COA}^{TGT} = (R^{SCP})_{COA}^{TGT} + \Delta R_{COA}^{TGT} \qquad Rdot_{COA}^{TGT} = (Rdot^{SCP})_{COA}^{TGT} + \Delta Rdot_{COA}^{TGT}$$

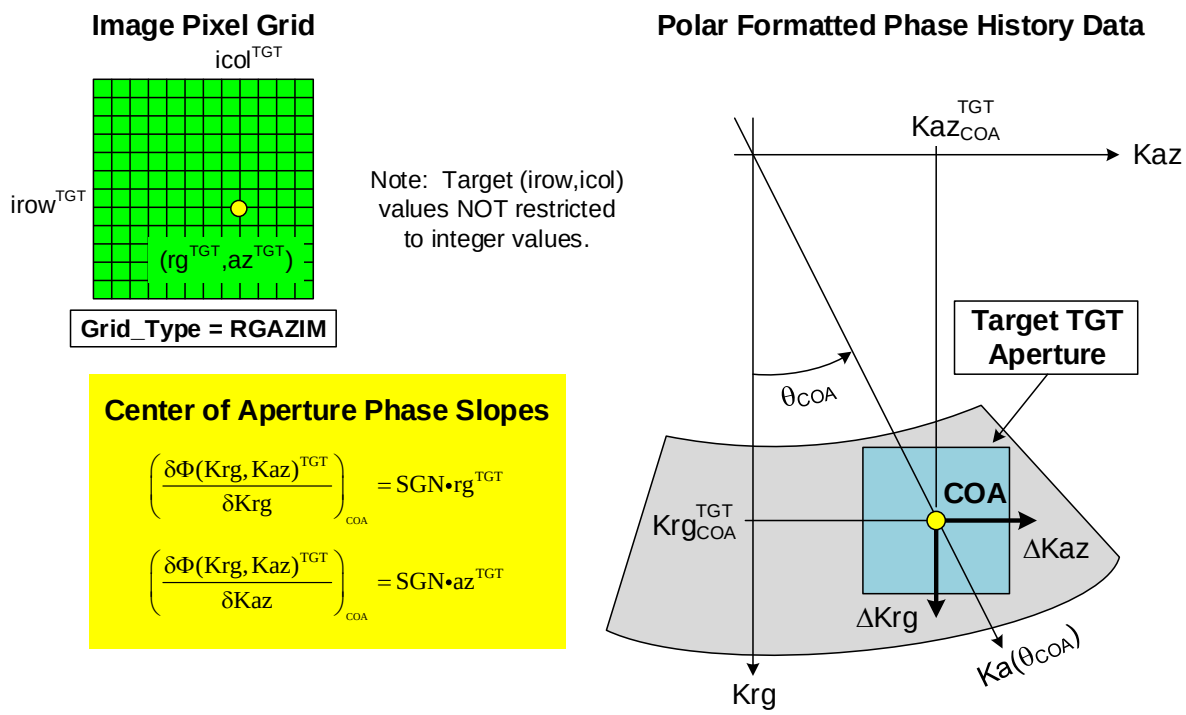
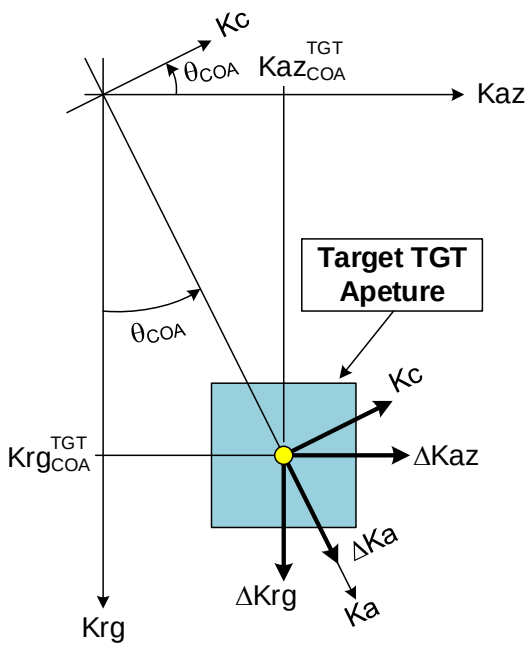


Figure 4-2 Image Pixel Grid & PFA Phase History Data

Phase slopes at the COA of the polar formatted PHD are proportional to rg^{TGT} and az^{TGT}.

Polar Formatted Phase History Data



Center of Aperture Phase Slopes

$$\left(\frac{\delta\Phi^{TGT}}{\delta Krg}\right)_{COA} = SGN \bullet rg^{TGT}$$

$$\left(\frac{\delta\Phi^{TGT}}{\delta Kaz}\right)_{COA} = SGN \bullet az^{TGT}$$

$$\left(\frac{\delta\Phi^{TGT}}{\delta Ka}\right)_{COA} = \cos(\theta_{COA}) \bullet \left(\frac{\delta\Phi^{TGT}}{\delta Krg}\right)_{COA} + \sin(\theta_{COA}) \bullet \left(\frac{\delta\Phi^{TGT}}{\delta Kaz}\right)_{COA}$$

$$\left(\frac{\delta\Phi^{TGT}}{\delta Kc}\right)_{COA} = -\sin(\theta_{COA}) \bullet \left(\frac{\delta\Phi^{TGT}}{\delta Krg}\right)_{COA} + \cos(\theta_{COA}) \bullet \left(\frac{\delta\Phi^{TGT}}{\delta Kaz}\right)_{COA}$$

Figure 4-3 Saved Aperture & Phase Slopes At COA

Phase slopes at COA of the polar formatted PHD.

4.2 Image Grid To R/Rdot: Grid_Type = RGAZIM & IFA = RGAZCOMP

For the RGAZIM grid, the image coordinates are range and azimuth. The row coordinate is the range coordinate, xrow = rg. The column coordinate is the azimuth coordinate, ycol = az. For image pixel location (irow, icol), the (rg, az) coordinates are computed as follows.

$$\mathrm{rg} = \mathrm{xrow} = \mathrm{Row_SS} \bullet \mathrm{irow} \qquad \mathrm{az} = \mathrm{ycol} = \mathrm{Col_SS} \bullet \mathrm{icol}$$

For images formed with the Range & Azimuth Compression algorithm, the R/Rdot computation makes use of the following SICD RGAZCOMP specific parameter.

RGAZCOMP Parameter:

AzSF	Scale factor that converts azimuth coordinate to an increment in cosine of the DCA at COA.

For an image formed with simple Range & Azimuth Compression, all pixels have a common COA. The resulting image is sometimes referred to as a Range, Doppler image. The rows are samples at constant range at COA and the columns are samples at constant Doppler Cone Angle at COA. Refer to Figure 4-4.

RGAZCOMP Image Grid Location To R/Rdot Contour

(1) Consider a target (TGT) with impulse response centered at image grid location (irow^{TGT}, icol^{TGT}). Grid location (irow^{TGT}, icol^{TGT}) is converted to image coordinates (rg^{TGT}, az^{TGT}). The COA parameters are computed for image coordinates (rg^{TGT}, az^{TGT}).

$$(\mathrm{irow}^{TGT}, \mathrm{icol}^{TGT}) \;\rightarrow\; (\mathrm{rg}^{TGT}, \mathrm{az}^{TGT}) \;\rightarrow\; t_{COA}^{TGT} \;\rightarrow\; \mathbf{ARP}_{COA}^{TGT} \;\&\; \mathbf{VARP}_{COA}^{TGT}$$

(2) Compute the range and range rate to the SCP at COA.

$$R_{COA}^{SCP} = \left|\mathbf{ARP}_{COA}^{TGT} - \mathbf{SCP}\right| \qquad \mathrm{Rdot}_{COA}^{SCP} = \frac{1}{R_{COA}^{SCP}} \bullet \mathbf{VARP}_{COA}^{TGT} \bullet (\mathbf{ARP}_{COA}^{TGT} - \mathbf{SCP})$$

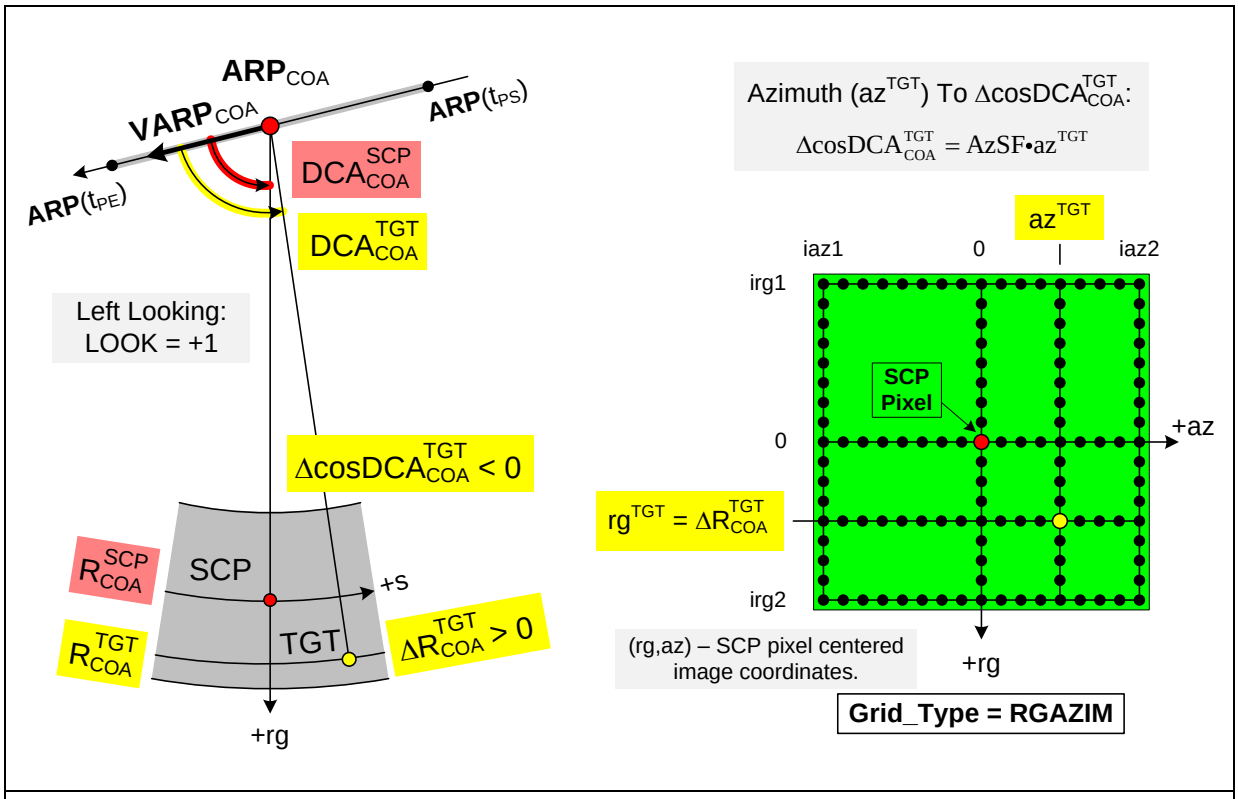


Figure 4-4 Image Formed With Range & Azimuth Compression
Rows are constant range at COA and columns are constant DCA at COA.

(3) Compute the increment in cosine of the DCA at COA of the target, $\Delta\text{cosDCA}_{\text{COA}}^{\text{TGT}}$, by scaling the azimuth coordinate by the azimuth to DCA scale factor. Compute the increment in range rate, $\Delta\text{Rdot}_{\text{COA}}^{\text{TGT}}$, by scaling by the magnitude of the velocity vector at COA.

$$\Delta\text{cosDCA}_{\text{COA}}^{\text{TGT}} = \text{AzSF}\bullet\text{az}^{\text{TGT}} \qquad \Delta\text{Rdot}_{\text{COA}}^{\text{TGT}} = -\left|\mathbf{VARP}_{\text{COA}}^{\text{TGT}}\right|\bullet\Delta\text{cosDCA}_{\text{COA}}^{\text{TGT}}$$

(4) Compute the range and range rate to the target at COA as follows.

$$\text{R}_{\text{COA}}^{\text{TGT}} = \text{R}_{\text{COA}}^{\text{SCP}} + \text{rg}^{\text{TGT}} \qquad \text{Rdot}_{\text{COA}}^{\text{TGT}} = \text{Rdot}_{\text{COA}}^{\text{SCP}} + \Delta\text{Rdot}_{\text{COA}}^{\text{TGT}}$$

4.3 Image Grid To R/Rdot: Grid_Type = RGZERO

For the RGZERO grid, the image coordinates are range and azimuth. The row coordinate is the range coordinate, xrow = rg. The column coordinates is the azimuth coordinate, ycol = az. For image pixel location (irow, icol), the (rg, az) coordinates are computed as follows.

$$rg = xrow = Row_SS \bullet irow \qquad\qquad az = ycol = Col_SS \bullet icol$$

For all images with Grid_Type = RGZERO, the following parameters are also included. The parameters describe the image formation processing with the RMA that formed the image.

RMA INCA Parameters:

cT_CA(n)	Time of Closest Approach polynomial coefficients. The polynomial yields time of closest approach, t_CA (sec), as a function of image azimuth coordinate (az). Time t = 0 at Collection Start.
N_TCA	Order of the Time of Closest Approach polynomial.
R_{CA}^{SCP}	Range at Closest Approach for the SCP (m).
cDRSF(m,n)	Doppler Rate Scale Factor polynomial coefficients. The polynomial yields the scale factor (DRSF) from image coordinates (rg,az). The scale factor is used to compute $d^2(R)/dt^2$ at closest approach.
M_DRSF	Order of the DRSF polynomial in the row/range coordinate.
N_DRSF	Order of the DRSF polynomial in the col/azimuth coordinate.

The computation of the R/Rdot contour for an arbitrary image grid location is presented with minimal explanation in order that the necessary computations are clearly presented. Refer to Figure 4-5. For a given RGZERO grid location, the range history that was processed to form the image signal is accurately described by range function $RA(t)^{TGT}$. The parameters of the $RA(t)^{TGT}$ are computed from the grid location. The range at COA is computed by evaluating the $RA(t)^{TGT}$ function at the t_{COA}^{TGT}. The range rate at COA is computed from the $d(RA(t)^{TGT})/dt$ at t_{COA}^{TGT}.

$$RA(t)^{TGT} = \left[(R_{CA}^{TGT})^2 + DRSF^{TGT} \bullet (VM_{CA}^{TGT})^2 \bullet (t - t_{CA}^{TGT})^2 \right]^{1/2}$$

RGZERO Image Grid Location To R/Rdot Contour

(1) Consider a target (TGT) with impulse response centered at image grid location ($irow^{TGT}$, $icol^{TGT}$). Grid location ($irow^{TGT}$, $icol^{TGT}$) is converted to image coordinates (rg^{TGT}, az^{TGT}). The COA parameters are computed for image coordinates (rg^{TGT}, az^{TGT}).

$$(\text{irow}^{TGT}, \text{icol}^{TGT}) \rightarrow (\text{rg}^{TGT}, \text{az}^{TGT}) \rightarrow t_{COA}^{TGT} \rightarrow \mathbf{ARP}_{COA}^{TGT} \ \& \ \mathbf{VARP}_{COA}^{TGT}$$

(2) Compute the range at closest approach and the time of closest approach for the image grid location. The range at closest approach, R_{CA}^{TGT}, is computed from the range coordinate. The time of closest approach, t_{CA}^{TGT}, is computed from the azimuth coordinate.

$$R_{CA}^{TGT} = R_{CA}^{SCP} + \text{rg}^{TGT} \qquad t_{CA}^{TGT} = \sum_{n=0}^{N_TCA} \text{cT_CA}(n) \bullet (\text{az}^{TGT})^n$$

(2) Compute the ARP velocity at t_{CA}^{TGT}, $\mathbf{VARP}_{CA}^{TGT}$. Also compute the magnitude of the vector.

$$\mathbf{VARP}_{CA}^{TGT} = \begin{bmatrix} \text{VARPx}(t_{CA}^{TGT}) \\ \text{VARPy}(t_{CA}^{TGT}) \\ \text{VARPz}(t_{CA}^{TGT}) \end{bmatrix} \qquad \text{VM}_{CA}^{TGT} = \left| \mathbf{VARP}_{CA}^{TGT} \right|$$

(3) Compute the Doppler Rate Scale Factor, DRSF^{TGT}, for image grid location (rg^{TGT}, az^{TGT}).

$$\text{DRSF}^{TGT} = \sum_{m=0}^{M_DRSF} \sum_{n=0}^{N_DRSF} \text{cDRSF}(m,n) \bullet (\text{rg}^{TGT})^m \bullet (\text{az}^{TGT})^n$$

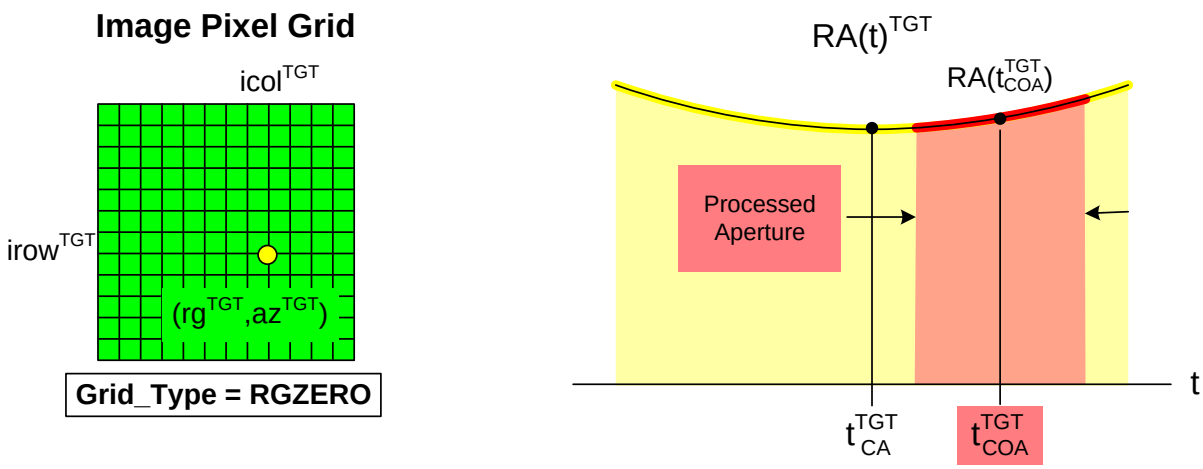
(4) Compute the time difference between the COA time and the CA time, Δt_{COA}^{TGT}.

$$\Delta t_{COA}^{TGT} = t_{COA}^{TGT} - t_{CA}^{TGT}$$

(5) Compute the range and range rate relative to the ARP at COA (R_{COA}^{TGT} and Rdot_{COA}^{TGT}). The projection to three-dimensional scene point for grid location (rg^{TGT}, az^{TGT}) is along this R/Rdot contour.

$$R_{COA}^{TGT} = \left[(R_{CA}^{TGT})^2 + \text{DRSF}^{TGT} \bullet (\text{VM}_{CA}^{TGT})^2 \bullet (\Delta t_{COA}^{TGT})^2 \right]^{1/2}$$

$$\text{Rdot}_{COA}^{TGT} = \frac{\text{DRSF}^{TGT}}{R_{COA}^{TGT}} \bullet (\text{VM}_{CA}^{TGT})^2 \bullet \Delta t_{COA}^{TGT}$$



$$RA(t)^{TGT} = \left[(R_{CA}^{TGT})^2 + DRSF^{TGT} \bullet (VM_{CA}^{TGT})^2 \bullet (t - t_{CA}^{TGT})^2 \right]^{1/2}$$

$$R_{COA}^{TGT} = RA(t)^{TGT}\Big|_{t=t_{COA}^{TGT}} \qquad Rdot_{COA}^{TGT} = \frac{d(RA(t)^{TGT})}{dt}\Bigg|_{t=t_{COA}^{TGT}}$$

Figure 4-5 Image Pixel Grid Type RGZERO

Pixel grid location specifies Time of Closest Approach and Range At Closest Approach.

4.4 Image Grid To R/Rdot: Grid_Type = XRGYCR

For the XRGYCR grid, the image coordinates are xrg and ycr. The xrg coordinate is the row coordinate, xrg = xrow. The ycr coordinates is the column coordinate, ycr = ycol. For image pixel location (irow, icol), the (xrg, ycr) coordinates are computed as follows.

$$\text{xrg} = \text{xrow} = \text{Row_SS} \bullet \text{irow} \qquad \text{ycr} = \text{ycol} = \text{Col_SS} \bullet \text{icol}$$

XRGYCR Image Grid Location To R/Rdot Contour

(1) Consider a target (TGT) with impulse response centered at image grid location (irow^{TGT}, icol^{TGT}). Grid location (irow^{TGT}, icol^{TGT}) is converted to image coordinates (xrg^{TGT}, ycr^{TGT}). The COA parameters are computed for image coordinates (xrg^{TGT}, ycr^{TGT}).

$$(\text{irow}^{TGT},\text{icol}^{TGT}) \;\rightarrow\; (\text{xrg}^{TGT},\text{ycr}^{TGT}) \;\rightarrow\; t_{COA}^{TGT} \;\rightarrow\; \mathbf{ARP}_{COA}^{TGT} \;\&\; \mathbf{VARP}_{COA}^{TGT}$$

(2) The samples of XRGYCR grid are uniformly spaced locations in the image plane formed by the SCP, and image plane vectors **uRow** and **uCol**. Vectors **uRow** and **uCol** are orthogonal. Compute the point the image plane point for image grid location (xrg^{TGT}, ycr^{TGT}).

$$\mathbf{IPP}^{TGT} = \mathbf{SCP} + \text{xrg}^{TGT} \bullet \mathbf{uRow} + \text{ycr}^{TGT} \bullet \mathbf{uCol}$$

(3) Compute the range and range rate relative to the ARP at COA (R_{COA}^{TGT} and Rdot_{COA}^{TGT}) for image plane point $\mathbf{IPP}^{TGT}$. The projection to three-dimensional scene point for grid location (xrg^{TGT}, ycr^{TGT}) is along this R/Rdot contour.

$$R_{COA}^{TGT} = \left|\mathbf{ARP}_{COA}^{TGT} - \mathbf{IPP}^{TGT}\right| \qquad \text{Rdot}_{COA}^{TGT} = \frac{1}{R_{COA}^{TGT}} \bullet \mathbf{VARP}_{COA}^{TGT} \bullet (\mathbf{ARP}_{COA}^{TGT} - \mathbf{IPP}^{TGT})$$

4.5 Image Grid To R/Rdot: Grid_Type = XCTYAT

For the XCTYAT grid, the image coordinates are xct and yat. The xct coordinate is the row coordinate, xct = xrow. The yat coordinate is the column coordinate, yat = ycol. The samples of the XCTYAT grid are uniformly spaced locations in the image plane (similar to the XRGYCR grid). The computation from image grid location to R/Rdot contour is the same as the shown in Section 4.4 above. The equations are listed below without further description.

(1) $(\mathrm{irow}^{TGT}, \mathrm{icol}^{TGT}) \rightarrow (\mathrm{xct}^{TGT}, \mathrm{yat}^{TGT}) \rightarrow t_{COA}^{TGT} \rightarrow \mathbf{ARP}_{COA}^{TGT}\ \&\ \mathbf{VARP}_{COA}^{TGT}$

(2) $\mathbf{IPP}^{TGT} = \mathbf{SCP} + \mathrm{xct}^{TGT} \bullet \mathbf{uRow} + \mathrm{yat}^{TGT} \bullet \mathbf{uCol}$

(3) $R_{COA}^{TGT} = \left| \mathbf{ARP}_{COA}^{TGT} - \mathbf{IPP}^{TGT} \right| \qquad \mathrm{Rdot}_{COA}^{TGT} = \frac{1}{R_{COA}^{TGT}} \bullet \mathbf{VARP}_{COA}^{TGT} \bullet (\mathbf{ARP}_{COA}^{TGT} - \mathbf{IPP}^{TGT})$

4.6 Image Grid To R/Rdot: Grid_Type = PLANE

For the PLANE grid, the image coordinates are xrow and ycol. The samples of the XCTYAT grid are uniformly spaced locations in the image plane (similar to the XRGYCR grid). Image plane vectors **uRow** and **uCol** may or may not be orthogonal. The computation from image grid location to R/Rdot contour is the same as the shown in Section 4.4 above. The equations are listed below without further description.

(1) $(\mathrm{irow}^{TGT}, \mathrm{icol}^{TGT}) \rightarrow (\mathrm{xrow}^{TGT}, \mathrm{ycol}^{TGT}) \rightarrow t_{COA}^{TGT} \rightarrow \mathbf{ARP}_{COA}^{TGT}\ \&\ \mathbf{VARP}_{COA}^{TGT}$

(2) $\mathbf{IPP}^{TGT} = \mathbf{SCP} + \mathrm{xrow}^{TGT} \bullet \mathbf{uRow} + \mathrm{ycol}^{TGT} \bullet \mathbf{uCol}$

(3) $R_{COA}^{TGT} = \left| \mathbf{ARP}_{COA}^{TGT} - \mathbf{IPP}^{TGT} \right| \qquad \mathrm{Rdot}_{COA}^{TGT} = \frac{1}{R_{COA}^{TGT}} \bullet \mathbf{VARP}_{COA}^{TGT} \bullet (\mathbf{ARP}_{COA}^{TGT} - \mathbf{IPP}^{TGT})$

5 Precise R/Rdot To Ground Plane Projection

The precise projection to a ground plane along an R/Rdot contour is described. The R/Rdot contour is relative to an ARP Center Of Aperture position and velocity.

5.1 Ground Plane & Ground Plane Normal

The ground plane is defined by a reference point in the plane, GREF, and the vector normal to the plane, **GPN**. The reference point and plane orientation may be based upon specific terrain height and slope information for the imaged area. When only a reference point is specified, a ground plane normal may be derived in several ways. Given the position of GREF in ECF coordinates, **GREF**, two ground planes that contain point GREF are: (1) the Spherical Earth ground plane and (2) the Geodetic Earth ground plane. The Spherical Earth ground plane is tangent to a spherical earth model passing through GREF. The Spherical Earth GP unit normal is computed as follows.

Spherical Earth GPN: $$\mathbf{uGPN}_{\text{SPH}} = \frac{1}{|\mathbf{GREF}|} \bullet \mathbf{GREF}$$

The Geodetic Earth ground plane is tangent to the surface of constant geodetic height above the WGS-84 reference ellipsoid passing through GREF. The Geodetic Earth GP unit normal is computed as follows by first computing the geodetic latitude and longitude of GREF.

$$[\text{GREF.X, GREF.Y, GREF.Z}] \rightarrow \boxed{\begin{array}{c}\text{Compute Geodetic}\\ \text{Latitude \& Longitude}\end{array}} \rightarrow [\text{GREF.Lat, GREF.Lon}]$$

Geodetic Earth GPN: $$\mathbf{uGPN}_{\text{GEO}} = \begin{bmatrix} \cos(\text{GREF.Lat}) \bullet \cos(\text{GREF.Lon}) \\ \cos(\text{GREF.Lat}) \bullet \sin(\text{GREF.Lon}) \\ \sin(\text{GREF.Lat}) \end{bmatrix}$$

5.2 R/Rdot Contour Ground Plane Intersection

For target TGT with COA range and range rate $R_{\text{COA}}^{\text{TGT}}$ and $\text{Rdot}_{\text{COA}}^{\text{TGT}}$, solve for the intersection of the COA contour and a ground plane. The R/Rdot contour is relative to an ARP position and velocity at COA, $\mathbf{ARP}_{\text{COA}}^{\text{TGT}}$ and $\mathbf{VARP}_{\text{COA}}^{\text{TGT}}$. The ground plane is defined by a point in the plane (GREF) and a vector normal to the plane. The intersection of the R/Rdot contour and the ground plane is point GPP^{TGT}. See Figure 5-1.

The computation makes use of a Cartesian coordinate system (XYZ) with origin at the ARP ground plane nadir (AGPN). The X and Y axes are in the ground plane and the Z axis is normal to it. The +X direction is in the Along Track direction. The unit Line of Sight

(**uLOS**) vector is defined by the azimuth angle (θ_{AZ}) and the grazing angle (Ψ_{GRAZ}). The range to the ground point determines the grazing angle. The range rate determines the cosine of the azimuth angle. The Side of Track determines the sign of the azimuth angle. The expression for the range rate with the velocity vector and the unit LOS vector in XYZ coordinates is used to solve for the cosine of the azimuth angle.

$$\mathrm{Rdot}_{\mathrm{COA}}^{\mathrm{TGT}} = -\mathbf{VARP}_{\mathrm{COA}}^{\mathrm{TGT}} \bullet \mathbf{uLOS} = -\begin{bmatrix} \mathrm{Vx} \\ 0 \\ \mathrm{Vz} \end{bmatrix} \bullet \begin{bmatrix} \cos(\Psi_{\mathrm{GRAZ}}) \bullet \cos(\theta_{\mathrm{AZ}}) \\ \cos(\Psi_{\mathrm{GRAZ}}) \bullet \sin(\theta_{\mathrm{AZ}}) \\ -\sin(\Psi_{\mathrm{GRAZ}}) \end{bmatrix}$$

$$\mathrm{Rdot}_{\mathrm{COA}}^{\mathrm{TGT}} = -\mathrm{Vx} \bullet \cos(\Psi_{\mathrm{GRAZ}}) \bullet \cos(\theta_{\mathrm{AZ}}) + \mathrm{Vz} \bullet \sin(\Psi_{\mathrm{GRAZ}})$$

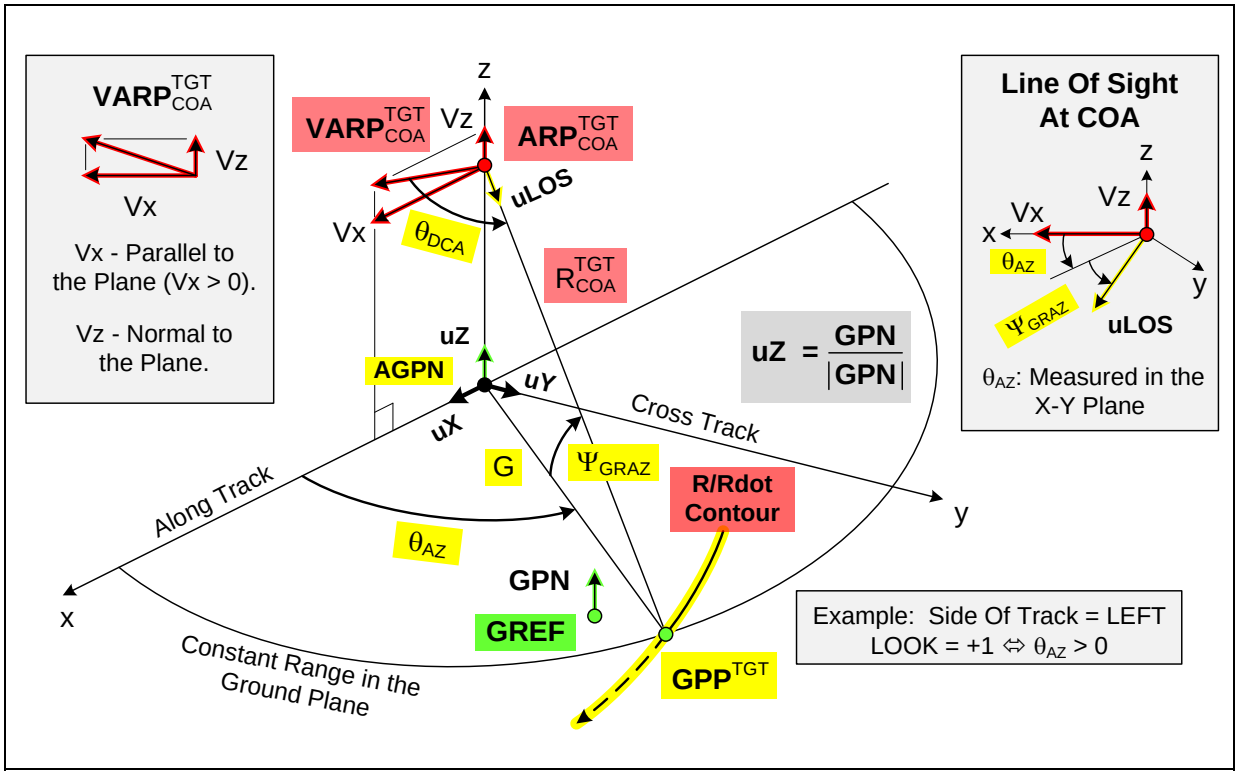


Figure 5-1 Projection Along R/Rdot Contour
Ground Plane Point (GPP) at range R_{COA} and range rate $Rdot_{COA}$.

Input Parameters:

$\mathbf{ARP}_{\mathrm{COA}}^{\mathrm{TGT}}$	ARP position at the COA time in ECF coordinates.
$\mathbf{VARP}_{\mathrm{COA}}^{\mathrm{TGT}}$	ARP velocity at the COA time in ECF coordinates.
LOOK	Integer based on Side of Track parameter.
	SideOfTrack = L ⇔ LOOK = +1 SideOfTrack = R ⇔ LOOK = -1

R_{COA}^{TGT}	Range of the R/Rdot contour.
$Rdot_{COA}^{TGT}$	Range rate (dR/dt) of the R/Rdot contour ($Rdot_{COA}^{TGT} < 0 \Leftrightarrow$ Closing geometry).
GREF	Ground plane reference point in ECF coordinates.
GPN	Ground plane normal vector in ECF coordinates.

Computed Parameters:

$\mathbf{GPP}^{TGT}$	Ground Plane Point (GPP^{TGT}) position in the ground plane and along the R/Rdot contour.

All position and velocity vectors are in ECF coordinates but may be in any common coordinate frame.

R/Rdot Contour To Ground Plane Point GPP

(1) Compute the unit vector in the +Z direction (normal to the ground plane).

$$\mathbf{uZ} = \frac{1}{|\mathbf{GPN}|} \bullet \mathbf{GPN}$$

(2) Compute the ARP distance from the plane (ARPz). Also compute the ARP ground plane nadir (**AGPN**).

$$ARPz = (\mathbf{ARP}_{COA}^{TGT} - \mathbf{GREF}) \bullet \mathbf{uZ}$$

Note: For $|ARPz| > R_{COA}^{TGT}$ ➔ No Solution

$$\mathbf{AGPN} = \mathbf{ARP}_{COA}^{TGT} - ARPz \bullet \mathbf{uZ}$$

(3) Compute the ground plane distance (G) from the ARP nadir to the circle of constant range. Also compute the sine and cosine of the grazing angle (Ψ_{GRAZ}).

$$G = +\left((R_{COA}^{TGT})^2 - ARPz^2\right)^{1/2} \quad cos_GRAZ = \frac{G}{R_{COA}^{TGT}} \quad sin_GRAZ = \frac{ARPz}{R_{COA}^{TGT}}$$

(4) Compute velocity components normal the ground plane (Vz) and parallel to the ground plane (Vx).

$$Vz = \mathbf{VARP}_{COA}^{TGT} \bullet \mathbf{uZ} \qquad VMag = \left|\mathbf{VARP}_{COA}^{TGT}\right|$$

$$Vx = +(VMag^2 - Vz^2)^{1/2}$$

Note: For Vx = 0 ➔ No Solution

(5) Orient the +X direction in the ground plane such that the Vx > 0. Compute unit vectors **uX** and **uY**.

$$\mathbf{uX} = \frac{1}{\mathrm{Vx}} \bullet (\mathbf{VARP}_{\mathrm{COA}}^{\mathrm{TGT}} - \mathrm{Vz} \bullet \mathbf{uZ}) \qquad \mathbf{uY} = \mathbf{uZ} \times \mathbf{uX}$$

(6) Compute the cosine of the azimuth angle (θ_{AZ}) to the ground plane point.

$$\mathrm{cos_AZ} = \frac{-\mathrm{Rdot}_{\mathrm{COA}}^{\mathrm{TGT}} + \mathrm{Vz} \bullet \mathrm{sin_GRAZ}}{\mathrm{Vx} \bullet \mathrm{cos_GRAZ}}$$

Note: For $\mathrm{cos_AZ} < -1.0$ ➔ No Solution. $\mathrm{Rdot}_{\mathrm{COA}}^{\mathrm{TGT}}$ > Max{Rdot} at ground distance G.

Note: For $\mathrm{cos_AZ} > 1.0$ ➔ No Solution. $\mathrm{Rdot}_{\mathrm{COA}}^{\mathrm{TGT}}$ < Min{Rdot} at ground distance G.

(7) Compute the sine of the azimuth angle. Use parameter LOOK to establish the correct sign corresponding to the correct Side of Track.

$$\mathrm{sin_AZ} = \mathrm{LOOK} \bullet (1 - \mathrm{cos_AZ}^2)^{1/2}$$

(8) Compute $\mathbf{GPP}^{\mathrm{TGT}}$ at distance G from the AGPN and at the correct azimuth angle.

$$\mathbf{GPP}^{\mathrm{TGT}} = \mathbf{AGPN} + \mathrm{G} \bullet \mathrm{cos_AZ} \bullet \mathbf{uX} + \mathrm{G} \bullet \mathrm{sin_AZ} \bullet \mathbf{uY}$$

6 Scene To Image Grid Projection

The precise Scene Point to Image Grid projection maps a geolocated point in the three-dimensional scene to a single image grid location. The computed image grid location is the center of the two-dimensional image signal response of a fixed point scatterer located at the scene point. The image grid location is computed using an iterative approach. The iterative approach includes many of the computations described in Sections 2 through 5. Only scene points on the correct Side of Track and at ranges and angles consistent with the imaging collection will yield meaningful results.

For projecting a single scene point to the image grid, the iterative approach is a simple computational task. The approach yields sub-pixel accuracy in a small number of iterations (typically in 2 iterations for ground points at the same height as the SCP). The projection of a single scene point to the image grid is described in Section 6.1. The precise resampling of an entire image to a geolocated surface requires the image grid location to be computed for many scene points. For precise resampling to a complex surface such as a terrain height model of a mountainous scene, all scene points may require the iterative approach. For precise resampling to a smooth surface such as a ground plane, the iterative approach may be applied to only a fraction of the scene points. An approach for resampling to smooth surfaces is described in Section 6.2.

6.1 Scene To Image: Single Scene Point

Let point S be the point in the imaged scene for which the image grid location is to be computed. Define a ground plane that contains point S. The iterative approach computes a sequence of ground plane points $G_1, G_2, \ldots, G_N$. The ground plane points $\{G_n\}$ are projected along straight lines to the Image Plane to produce a sequence of image grid points $I_1, I_2, \ldots, I_N$. Image point I_N is the final image grid location and is the precise projection of point S. For each image plane point I_n, the precise R/Rdot projection to the ground plane is computed. The precise ground plane projection of image point I_n is point P_n. Final point I_N is established when the displacement between point S and point P_N is less than a user specified tolerance. When the displacement between point S and point P_n is greater than the tolerance, the displacement is added to point G_n to yield point G_{n+1} for the next iteration. An example projection sequence is shown in Figure 6-1. In the example shown, convergence is achieved in 3 iterations. Image grid location ($xrow_3$, $ycol_3$) is accepted as the precise projection of scene point S.

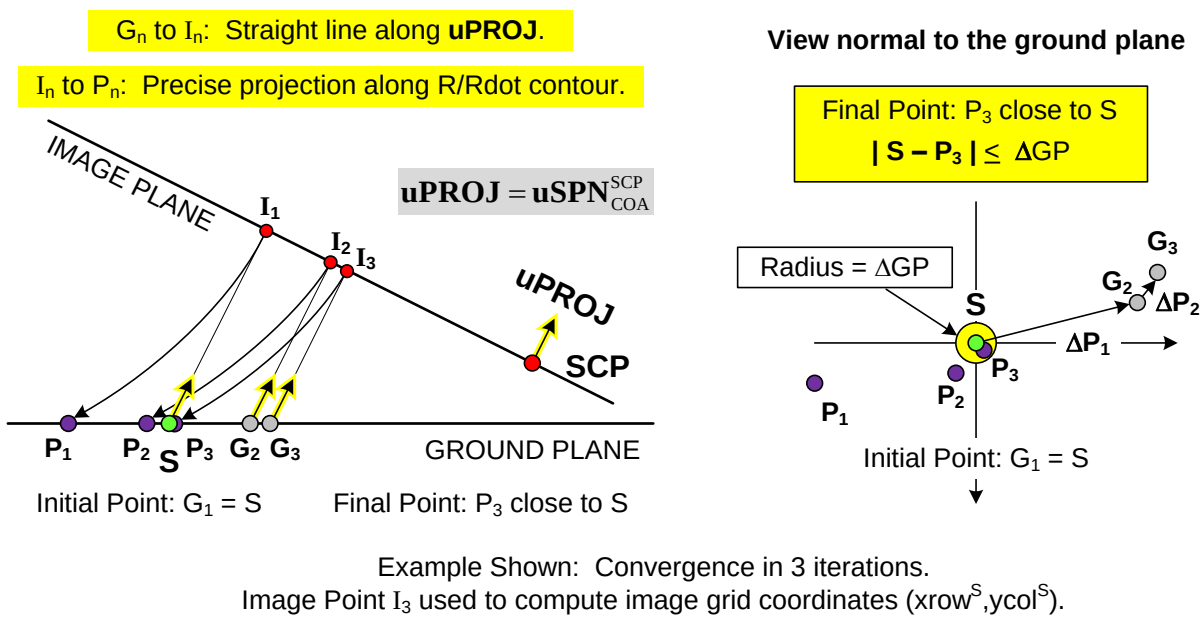


Figure 6-1 Scene To Image Iteration
Example Scene to Image computation converging in 3 iterations.

Scene Point S To Image Grid ($xrow^S$, $ycol^S$)

(1) For scene point **S**, compute the spherical earth ground plane unit normal. The choice of ground plane containing point S is not critical. See Section 5.1. Also, choose the ground plane displacement threshold, ΔGP_MAX, for final ground plane point P_N.

$$\mathbf{uGPN} = \frac{1}{|\mathbf{S}|} \bullet \mathbf{S} \qquad \Delta GP_MAX = \text{Maximum Ground Plane Displacement}$$

(2) Ground plane points $\{G_n\}$are projected along straight lines to the image plane to establish points $\{I_n\}$. The GP to IP projection direction is along the SCP COA slant plane normal. Also, compute the image plane unit normal, **uIPN**. Compute projection scale factor SF as shown.

$$\mathbf{uPROJ} = \mathbf{uSPN}^{SCP}_{COA} \qquad \mathbf{IPN} = \mathbf{uRow} \times \mathbf{uCol} \qquad \mathbf{uIPN} = \frac{1}{|\mathbf{IPN}|} \bullet \mathbf{IPN}$$

$$SF = \mathbf{uPROJ} \bullet \mathbf{uIPN}$$

(3) Set initial ground plane position G_1 to the scene point position S.

$$\mathbf{G}_1 = \mathbf{S} \qquad \text{Initial ground plane position, } n = 1.$$

Iterate over steps (4), (5) and (6). For ground point G_n, compute image plane point I_n and ground plane point P_n.

(4) Project ground plane point G_n to image plane point I_n. The projection distance is $DIST_n$. Compute image coordinates $xrow_n$ and $ycol_n$. See Section 2.4.

$$\mathrm{DIST}_n = \frac{1}{\mathrm{SF}} \bullet (\mathbf{SCP} - \mathbf{G}_n) \bullet \mathbf{uIPN} \qquad \mathbf{I}_n = \mathbf{G}_n + \mathrm{DIST}_n \bullet \mathbf{uPROJ}$$

$$\mathbf{I}_n \rightarrow \boxed{\begin{array}{c}\text{Compute}\\ \text{Image}\\ \text{Coordinates}\end{array}} \rightarrow \begin{array}{c}\mathrm{xrow}_n\\ \mathrm{ycol}_n\end{array}$$

(5) Compute the precise projection for image grid location ($xrow_n$, $ycol_n$) to the ground plane containing the scene point S. The result is point P_n. For image grid location ($xrow_n$, $ycol_n$), compute COA parameters per Section 2. Compute the precise R/Rdot projection contour per Section 4. Compute the R/Rdot intersection with the ground plane per Section 5.

$$\begin{array}{c}\mathrm{xrow}_n\\ \mathrm{ycol}_n\end{array} \rightarrow \boxed{\begin{array}{c}\text{Compute}\\ \text{COA}\\ \text{Parameters}\end{array}} \rightarrow \boxed{\begin{array}{c}\text{Compute}\\ \text{R/Rdot}\\ \text{Contour}\end{array}} \rightarrow \boxed{\begin{array}{c}\text{Project}\\ \text{Along R/Rdot}\\ \text{To GP}\end{array}} \rightarrow \mathbf{P}_n$$

(6) Compute the displacement between ground plane point P_n and the scene point S. If the displacement is greater than the threshold (ΔGP_MAX), compute point G_{n+1} and repeat the projections in steps (4) and (5) above. If the displacement is less than the threshold, accept image grid location ($xrow_n$, $ycol_n$) as the precise image grid location for scene point S.

$$\Delta\mathbf{P}_n = \mathbf{S} - \mathbf{P}_n \qquad \Delta\mathrm{GP}_n = \left|\Delta\mathbf{P}_n\right|$$

If $\Delta\mathrm{GP}_n > \Delta\mathrm{GP_MAX}$:

$\mathbf{G}_{n+1} = \mathbf{G}_n + \Delta\mathbf{P}_n$ Compute points I_{n+1} and P_{n+1}.

If $\Delta\mathrm{GP}_n \leq \Delta\mathrm{GP_MAX}$:

$\mathrm{xrow}^S = \mathrm{xrow}_n \qquad \mathrm{ycol}^S = \mathrm{ycol}_n$

The precise projection may yield an image grid location that is off the image pixel grid. Such a solution is valid and indicates that the scene point is not included in the image product.

6.2 Scene To Image: Smooth Surface

The precise resampling to a uniformly spaced grid on any geolocated surface may be accomplished by applying the iterative approach described in Section 6.1 to all scene points. However, for smooth surfaces the precise iterative approach may be applied to a subset of the scene points. The approach makes use of the fact that for smooth surfaces, the two-dimensional image grid location varies smoothly across the two-dimensional surface grid.

An example image grid projected to a smooth geolocated surface is shown in Figure 6-2. The smooth surface is at constant height above to the WGS-84 reference ellipsoid. The surface grid points are evenly spaced in latitude and longitude. The extent of the region covered by the image is bounded by corner points (Lat_0, Lon_0) and (Lat_1,Lon_1). These corner points are established by the precise image to scene projection of the image grid corner points to the constant height surface. Section 9 describes the projection. In the example shown, a sparse grid of 6 x 6 surface points is selected. The precise image grid location for each of the 36 points is computed using the iterative approach described above. The resulting image coordinates are saved for all grid points (including grid points that are not covered by the image). A dense set of image grid coordinates may be rapidly computed by bilinear interpolation of the corner point values. The accuracy of the interpolated image grid coordinates can be determined by the precise scene to image projection of the 5 x 5 set of scene points centered in cells formed by the 6 x 6 grid.

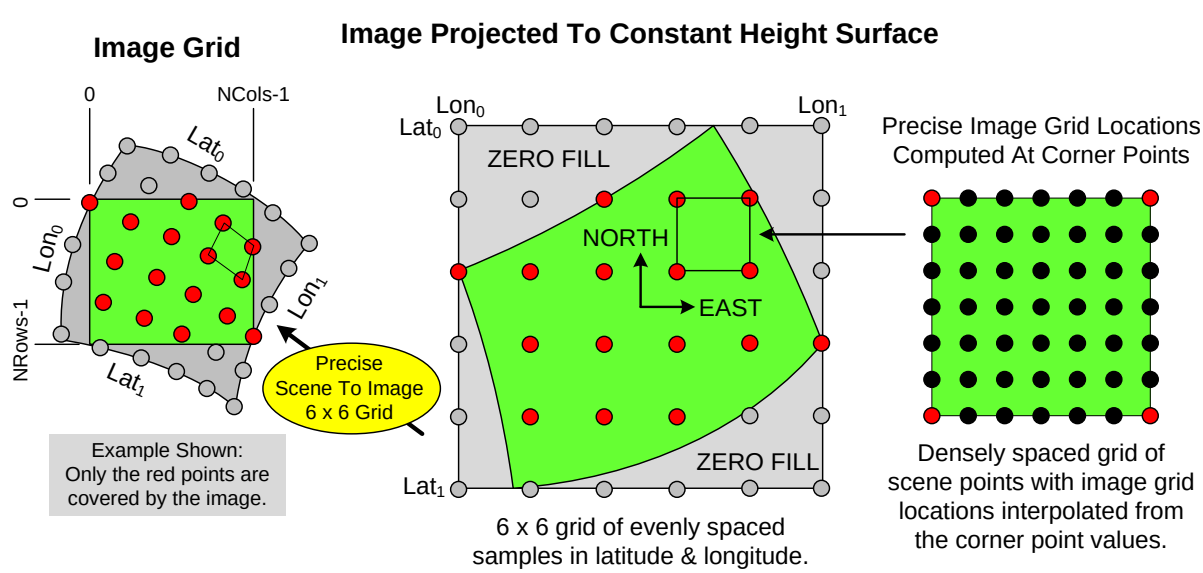


Figure 6-2 Image Projected To Smooth Surface
Precise Scene to Image computed for sparse grid and then interpolated.

7 Simple Ground Plane Projection

Precise ground plane resampling for large images can be a computational burden. For some applications, a fast but slightly less accurate algorithm is acceptable. A simple method for fast ground plane resampling is described. The fast projection algorithm uses the precise projection of the SCP pixel to the ground plane. For image grid locations displaced from the SCP pixel, a simple computation is used to estimate a ground plane offset from the projected SCP location. The simple projection is highly accurate for pixels near the SCP pixel. Accuracy degrades slowly with displacement from the SCP pixel. The fast algorithm may be applied to any image product independent of the image formation algorithm used to form the image. The simple projection can be used for projection to an arbitrary ground plane.

The simple projection is shown in Figure 7-1. For an arbitrary ground plane, the precise projection of the SCP pixel to the ground plane is computed. The resulting ground plane point is the Ground Center Point (GCP). A pair of orthogonal ground plane axes (GPX, GPY) are defined. For ground plane point GPP at location (GPX^{GPP}, GPY^{GPP}), image coordinates ($xrow^{GPP}$, $ycol^{GPP}$) are computed from a 2x2 projection matrix. Image indices ($irow^{GPP}$, $icol^{GPP}$) are computed from the image coordinates ($xrow^{GPP}$, $ycol^{GPP}$) by scaling by the sample spacings.

$$\begin{bmatrix} xrow^{GPP} \\ ycol^{GPP} \end{bmatrix} = \mathbf{M}_{GP}^{IC} \bullet \begin{bmatrix} GPX^{GPP} \\ GPY^{GPP} \end{bmatrix} = \begin{bmatrix} GPXtoRow & GPYtoRow \\ GPXtoCol & GPYtoCol \end{bmatrix} \bullet \begin{bmatrix} GPX^{GPP} \\ GPY^{GPP} \end{bmatrix}$$

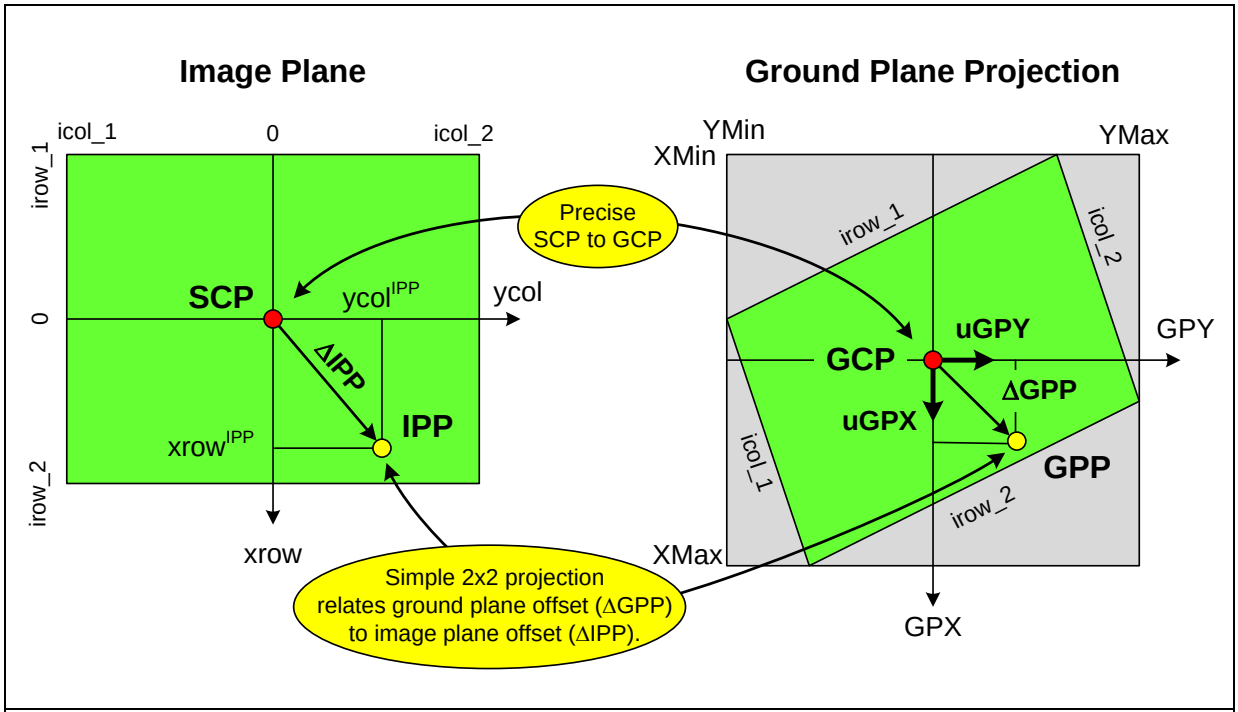


Figure 7-1 Simple Image Grid To Ground Plane
The projection from SCP pixel to GCP is precise.

7.1 Ground Plane Parameters

The ground plane is defined by a reference point in the plane (GREF) and the vector normal to the plane (GPN). Any arbitrary ground plane may be selected.

GREF = Ground plane reference point the ECF coordinates.

uGPN = Ground plane unit normal vector.

For many applications, a logical choice for the ground plane is the plane tangent to the surface of constant height above the ellipsoid at the SCP. See Section 5.1.

$$\mathbf{GREF} = \mathbf{SCP} \qquad \mathbf{uGPN} = \mathbf{uGPN}_{\text{GEO}}^{\text{SCP}} = \begin{bmatrix} \cos(\text{SCP.Lat})\bullet\cos(\text{SCP.Lon}) \\ \cos(\text{SCP.Lat})\bullet\sin(\text{SCP.Lon}) \\ \sin(\text{SCP.Lat}) \end{bmatrix}$$

The precise projection of the SCP pixel to the ground plane is computed. The SCP pixel COA parameters and R/Rdot projection contour are described in Section 3. The projection along an R/Rdot contour to a ground plane is described in Section 5. The result of the projection is the Ground Center Point (GCP). If the selected ground plane contains the SCP, the GCP position is set equal to the SCP position. See Figure 7-2. Ground points are projected along straight lines from the ground plane to the image plane. The ground plane to image plane projection direction is determined as follows. For the SCP in the ground plane, the projection direction is along the SCP COA slant plane normal. See Section 3. For the SCP not in the ground plane, the projection direction is from the GCP to the SCP.

For the SCP in the ground plane: $(\mathbf{SCP} - \mathbf{GREF}) \bullet \mathbf{uGPN} = 0$

$$\mathbf{GCP} = \mathbf{SCP} \qquad \mathbf{uPROJ} = \mathbf{uSPN}_{\text{COA}}^{\text{SCP}}$$

For the SCP not in ground plane: $(\mathbf{SCP} - \mathbf{GREF}) \bullet \mathbf{uGPN} \neq 0$

$$\boxed{\begin{array}{c}\text{Compute}\\ \text{SCP COA}\\ \text{Parameters}\end{array}} \rightarrow \boxed{\begin{array}{c}\text{Project}\\ \text{Along R/Rdot}\\ \text{To GP}\end{array}} \rightarrow \mathbf{GCP} \qquad \mathbf{uPROJ} = \frac{1}{|\mathbf{SCP} - \mathbf{GCP}|} \bullet (\mathbf{SCP} - \mathbf{GCP})$$

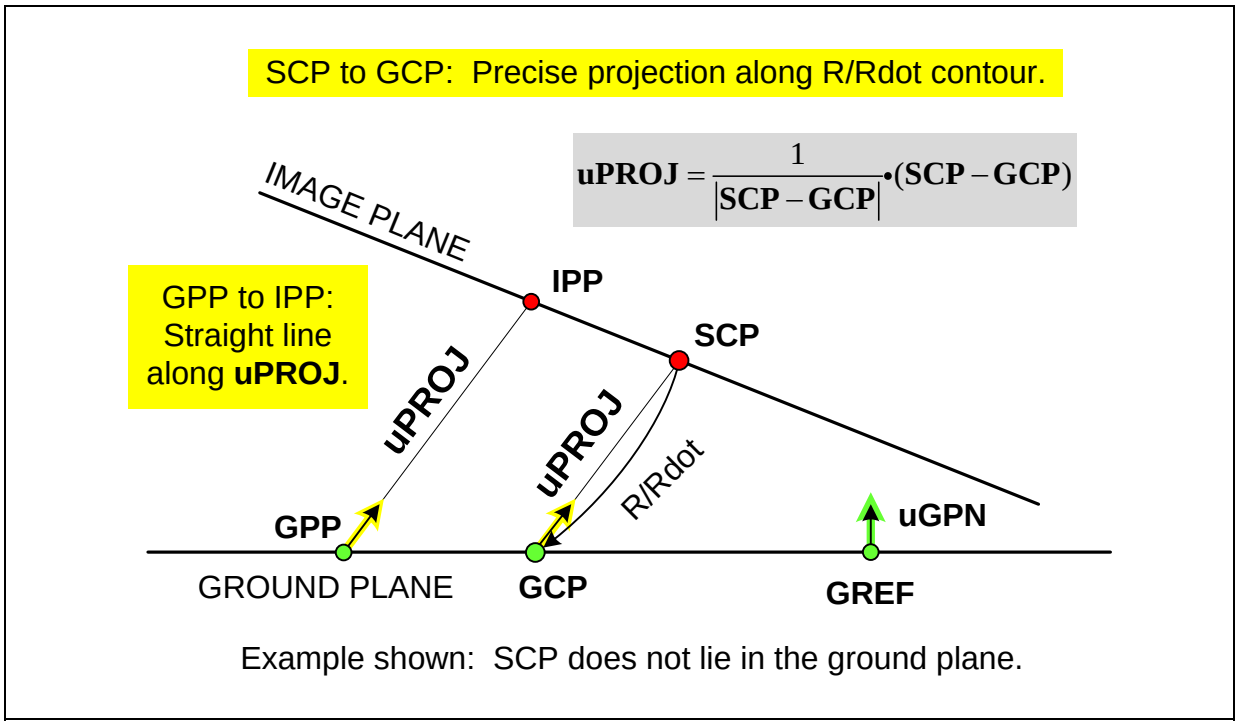


Figure 7-2 Straight Line Projection
Simple projection from Image Plane to the Ground plane along straight lines.

The orthogonal ground plane axes GPX and GPY may have any orientation and are defined by unit vectors **uGPX** and **uGPY**. The only restriction is that the vectors lie in the ground plane.

uGPX = Unit vector in the GPX direction in ECF coordinates.

uGPY = Unit vector in the GPY direction in ECF coordinates.

$$\mathbf{uGPX} \bullet \mathbf{uGPN} = 0 \qquad \mathbf{uGPY} \bullet \mathbf{uGPN} = 0 \qquad \mathbf{uGPX} \bullet \mathbf{uGPY} = 0$$

A logical choice of the GPX direction is in the shadow direction at the GCP. The shadow direction is from the ARP ground plane nadir (**AGPN**) to the **GCP**. For the ground plane image, GPX is in the increasing row direction and GPY is in the increasing column direction. For a shadows down orientation, compute unit vectors **uGPX** and **uGPY** as follows.

$$\mathbf{AGPN} = \mathbf{ARP}_{\mathrm{COA}}^{\mathrm{SCP}} - \left[(\mathbf{ARP}_{\mathrm{COA}}^{\mathrm{SCP}} - \mathbf{GCP}) \bullet \mathbf{uGPN} \right] \bullet \mathbf{uGPN}$$

$$\mathbf{uGPX} = \frac{1}{|\mathbf{GCP} - \mathbf{AGPN}|} \bullet (\mathbf{GCP} - \mathbf{AGPN})$$

$$\mathbf{uGPY} = \mathbf{uGPN} \times \mathbf{uGPX}$$

7.2 Ground Coordinates To Image Coordinates

The projection of a ground plane point to the image grid coordinates is computed as described below. For ground plane point GPP at location (GPX^{GPP}, GPY^{GPP}), the image coordinates are computed by projecting from the ground plane to the image plane in **uPROJ** direction. The resulting image plane point is IPP. The image plane parameters are described in Section 1.3.

uRow = Unit vector in the increasing row direction in ECF coordinates.

uCol = Unit vector in the increasing column direction in ECF coordinates.

θ_{COL} = Angle between unit vectors **uRow** and **uCol**.

uIPN = Image plane unit normal in ECF coordinates.

The projection of ground plane point GPP at location (GPX^{GPP}, GPY^{GPP}) to image plane position IPP located at image coordinates ($xrow^{GPP}$, $ycol^{GPP}$) is based on the following relationships.

GPP Position: $$\mathbf{GPP} = \mathbf{GCP} + \Delta\mathbf{GPP} = \mathbf{GCP} + \mathrm{GPX}\bullet\mathbf{uGPX} + \mathrm{GPY}\bullet\mathbf{uGPY}$$

Projection Distance: $$\mathrm{DIST} = \frac{(\mathbf{SCP} - \mathbf{GPP})\bullet\mathbf{uIPN}}{\mathbf{uPROJ}\bullet\mathbf{uIPN}}$$

IPP Position: $$\mathbf{IPP} = \mathbf{GPP} + \mathrm{DIST}\bullet\mathbf{uPROJ}$$

The ground coordinate to image coordinate projection matrix, $\mathbf{M}_{GP}^{IC}$, is computed as the product of two matrices. Matrix $\mathbf{M}_{GP}^{IV}$ is the projection of GPP to IPP and then orthogonal projection onto the two image plane vectors. Matrix $\mathbf{M}_{IV}^{IC}$ accounts for possible non-orthogonal image plane vectors.

$$\mathbf{M}_{GP}^{IC} = \begin{bmatrix} \mathrm{GPXtoRow} & \mathrm{GPYtoRow} \\ \mathrm{GPXtoCol} & \mathrm{GPYtoCol} \end{bmatrix} = \mathbf{M}_{IV}^{IC}\bullet\mathbf{M}_{GP}^{IV}$$

Compute the 2x2 matrix $\mathbf{M}_{GP}^{IV} = \begin{bmatrix} \mathrm{GPXtoRV} & \mathrm{GPYtoRV} \\ \mathrm{GPXtoCV} & \mathrm{GPYtoCV} \end{bmatrix}$:

$$\mathrm{GPXtoRV} = \mathbf{uGPX}\bullet\mathbf{uRow} - \frac{(\mathbf{uGPX}\bullet\mathbf{uIPN})\bullet(\mathbf{uPROJ}\bullet\mathbf{uRow})}{\mathbf{uPROJ}\bullet\mathbf{uIPN}}$$

$$\text{GPYtoRV} = \mathbf{uGPY} \bullet \mathbf{uRow} - \frac{(\mathbf{uGPY} \bullet \mathbf{uIPN}) \bullet (\mathbf{uPROJ} \bullet \mathbf{uRow})}{\mathbf{uPROJ} \bullet \mathbf{uIPN}}$$

$$\text{GPXtoCV} = \mathbf{uGPX} \bullet \mathbf{uCol} - \frac{(\mathbf{uGPX} \bullet \mathbf{uIPN}) \bullet (\mathbf{uPROJ} \bullet \mathbf{uCol})}{\mathbf{uPROJ} \bullet \mathbf{uIPN}}$$

$$\text{GPYtoCV} = \mathbf{uGPY} \bullet \mathbf{uCol} - \frac{(\mathbf{uGPY} \bullet \mathbf{uIPN}) \bullet (\mathbf{uPROJ} \bullet \mathbf{uCol})}{\mathbf{uPROJ} \bullet \mathbf{uIPN}}$$

Compute the 2x2 matrix $\mathbf{M}_{\text{IV}}^{\text{IC}}$:

$$\mathbf{M}_{\text{IV}}^{\text{IC}} = \frac{1}{\sin^2(\theta_{\text{COL}})} \bullet \begin{bmatrix} 1 & -\cos(\theta_{\text{COL}}) \\ -\cos(\theta_{\text{COL}}) & 1 \end{bmatrix}$$

Compute the 2x2 matrix $\mathbf{M}_{\text{GP}}^{\text{IC}} = \mathbf{M}_{\text{IV}}^{\text{IC}} \bullet \mathbf{M}_{\text{GP}}^{\text{IV}}$:

$$\mathbf{M}_{\text{GP}}^{\text{IC}} = \mathbf{M}_{\text{IV}}^{\text{IC}} \bullet \mathbf{M}_{\text{GP}}^{\text{IV}} = \frac{1}{\sin^2(\theta_{\text{COL}})} \bullet \begin{bmatrix} 1 & -\cos(\theta_{\text{COL}}) \\ -\cos(\theta_{\text{COL}}) & 1 \end{bmatrix} \bullet \begin{bmatrix} \text{GPXtoRV} & \text{GPYtoRV} \\ \text{GPXtoCV} & \text{GPYtoCV} \end{bmatrix}$$

8 Adjustable Parameters

The ARP position versus time and the range at COA may be adjusted in computing the Image To Scene and Scene To Image projections. The parameter adjustments are made by including the optional Adjustable Parameter Offsets in the image projection computations. The Adjustable Parameter Offsets are defined in Table 1-3 and are summarized below. Section 8.1 describes the use of the offsets in the Image To Scene projection. Section 8.2 describes the use of the offsets in Scene To Image projection.

Adjustable Parameters:

$\Delta\mathbf{ARP}_{COA}^{SCP}$	ARP position offset that is added to the computed ARP position at time $t = t_{COA}^{SCP}$.
$\Delta\mathbf{VARP}$	ARP velocity offset that is added to the computed ARP velocity (independent of time t).
ΔR_BIAS	Range bias offset that is added to the R_{COA} computed for any image grid location.

For a given image pixel location (xrow, ycol) with COA time t_{COA}, the Adjusted ARP COA position is the sum of the ARP COA position computed from the image metadata plus the ARP position offset, denoted $\Delta\mathbf{ARP}_{COA}$, also computed at t_{COA}. The Adjusted ARP position and velocity and the adjusted range at COA are computed as shown below.

$$\underbrace{\text{Adjusted } \mathbf{ARP}_{COA}}_{\substack{\text{Ideally Equal To The}\\ \text{True Position At Time } t_{COA}}} = \underbrace{\mathbf{ARP}_{COA}}_{\substack{\text{Computed from the}\\ \text{Image Metadata}}} + \underbrace{\Delta\mathbf{ARP}_{COA}^{SCP} + \Delta\mathbf{VARP}\bullet\left(t_{COA} - t_{COA}^{SCP}\right)}_{\substack{\Delta\mathbf{ARP}_{COA} = \text{ARP Offset}\\ \text{Added To Correct The Error}}}$$

$$\underbrace{\text{Adjusted } \mathbf{VARP}_{COA}}_{\substack{\text{Ideally Equal To The}\\ \text{True Velocity At Time } t_{COA}}} = \underbrace{\mathbf{VARP}_{COA}}_{\substack{\text{Computed from the}\\ \text{Image Metadata}}} + \underbrace{\Delta\mathbf{VARP}}_{\substack{\text{Constant Offset}\\ \text{For All Pixel Locations}}}$$

$$\underbrace{\text{Adjusted } R_{COA}(\text{xrow}, \text{ycol})}_{\substack{\text{Ideally Equal To The}\\ \text{True Range At COA}}} = \underbrace{R_{COA}(\text{xrow}, \text{ycol})}_{\substack{\text{Computed from the}\\ \text{Image Metadata}}} + \underbrace{\Delta\text{R_BIAS}}_{\substack{\text{Range Bias Offset}\\ \text{For All Pixel Locations}}}$$

Shown in Figure 8-1 is an example ARP trajectory before and after adjustment using the ARP position and velocity offsets. Also shown in the upper left portion of the figure is the image pixel array that was formed with COA times that vary with image pixel location. Shown are the pixel grid locations for three targets, denoted TGT A, TGT B and TGT C. The contours of constant COA time are also shown. The ARP COA positions along the trajectories are shown for each of the targets. For the example shown, the $\Delta\mathbf{ARP}_{COA}$ varies as the COA time varies across the image.

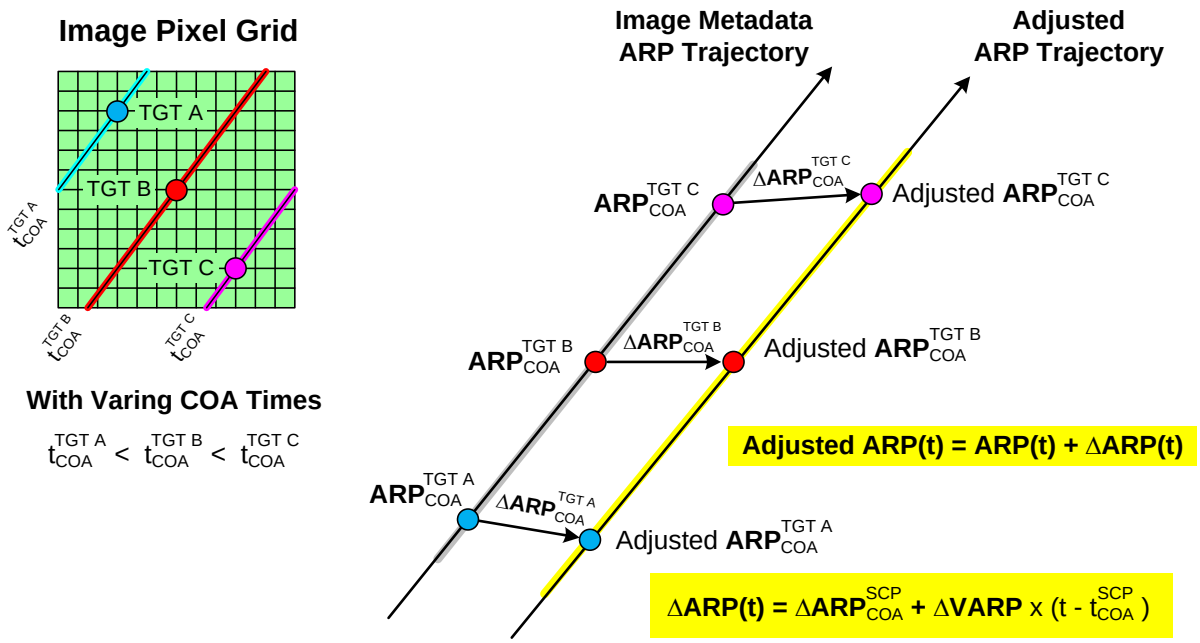


Figure 8-1 Original & Adjusted ARP Trajectories
The original ARP trajectory (from the image metadata) and the adjusted ARP trajectory.

8.1 Adjusted Image To Scene Projection

The Adjustable Parameter Offsets are incorporated into the computation from image pixel location to COA projection parameters. Figure 4-1 shows the computation of the initial COA projection set using only the image metadata. Image location (xrow, ycol) is used to compute the set of six parameters $\{t_{COA}, t^{SCP}_{COA}, \mathbf{ARP}_{COA}, \mathbf{VARP}_{COA}, R_{COA}$ and $Rdot_{COA}\}$. The Adjustable Parameter Offsets are then used to compute the Adjusted COA projection set. See Figure 8-2. Note that the ARP trajectory in the image metadata is used to compute the initial COA projection set. The Adjusted COA projection set is then used to project the R/Rdot contour to the ground surface model for the imaged scene.

For the SCP pixel location, the COA projection set computed from the image metadata will result in an R/Rdot contour that passes through SCP position. When the adjustable parameter offsets are applied, the resulting R/Rdot contour will, in general, no longer pass through the SCP position.

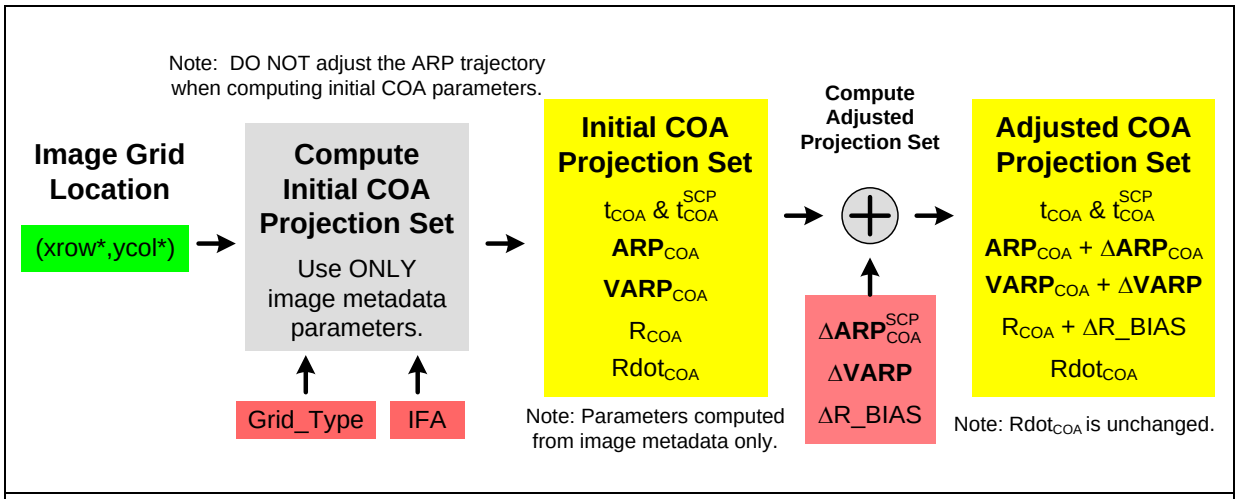
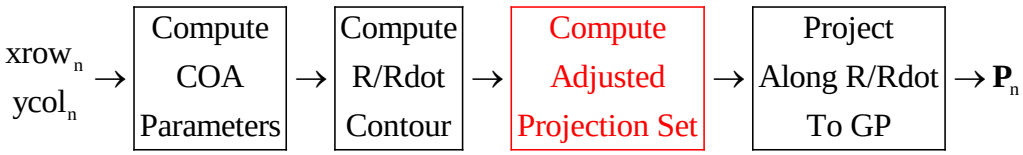


Figure 8-2 Use Of Adjustable Parameter Offsets
The offsets are added to the projection parameters computed from the image metadata.

8.2 Adjusted Scene To Image Projection

The precise Scene To Image projection computation is an iterative computation described in Section 6.1. The image pixel grid location to COA projection set computation is used once per pass of the iterative processing. See Step 5 in the description. The Adjustable Parameter Offsets are used to adjust the COA projection set. The adjusted COA projection set is then used to project to the ground plane containing the Scene Point. The modification to Step 5 is shown below.



9 Precise R/Rdot To Constant HAE Surface Projection

The precise projection to a surface of constant height above the WGS-84 reference ellipsoid along an R/Rdot contour is described. The R/Rdot contour is relative to an ARP Center Of Aperture position and velocity.

9.1 Constant Height Surface & Surface Normal

An example projection to the surface at constant height HAE_0 above the WGS-84 reference ellipsoid is shown is Figure 9-1. The projection along the R/Rdot contour yields surface point SPP^{TGT}. The surface point SPP^{TGT} position in geodetic coordinates is denoted ($SPP.Lat^{TGT}$, $SPP.Lon^{TGT}$, HAE_0). Also shown in Figure 9-1 is an arbitrary point S located on the constant height surface at geodetic coordinates (S.Lat, S.Lon, HAE_0). The unit normal vector to the surface at point S is $\mathbf{uUP}^S$. Vector $\mathbf{uUP}^S$ points in the direction of increasing HAE. The unit vector $\mathbf{uUP}^S$ in ECF coordinates is computed from the latitude and longitude at point S.

$$\mathbf{uUP}^S = \begin{bmatrix} \cos(S.Lat)\bullet\cos(S.Lon) \\ \cos(S.Lat)\bullet\sin(S.Lon) \\ \sin(S.Lat) \end{bmatrix}$$

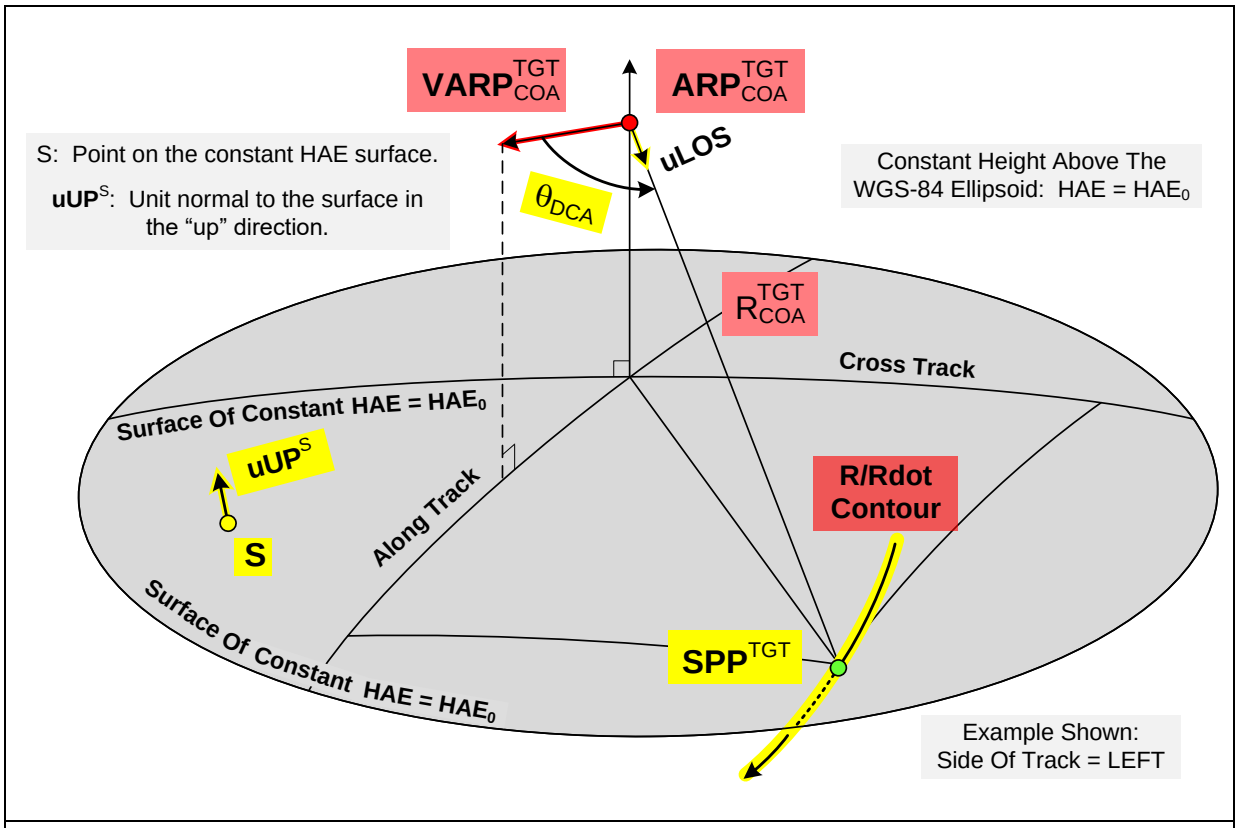


Figure 9-1 Projection To A Constant Height Surface
The surface point SPP^{TGT} is the precise projection to the surface of constant height HAE_0.

9.2 R/Rdot Contour To Surface Intersection

The projection of image pixel grid location ($xrow^{TGT}$, $ycol^{TGT}$) to a constant HAE surface is computed using an iterative algorithm. The algorithm computes the R/Rdot projection to one or more ground planes that are tangent to the constant height surface. Each ground plane projection point computed is slightly above the constant HAE surface. The final surface position SPP^{TGT} is computed by projecting from the final ground plane projection point down to the HAE surface.

Image Pixel Grid Location ($xrow^{TGT}$, $ycol^{TGT}$) COA Projection Set:

$\mathbf{ARP}_{COA}^{TGT}$	ARP position at the COA time in ECF coordinates. See Section 2.
$\mathbf{VARP}_{COA}^{TGT}$	ARP velocity at the COA time in ECF coordinates.
R_{COA}^{TGT}	Range of the R/Rdot contour. See Section 4.
$Rdot_{COA}^{TGT}$	Range rate (dR/dt) of the R/Rdot contour.

Image Parameters:

SCP	SCP position in ECF coordinates.
SCP.Lat	SCP geodetic latitude.
SCP.Lon	SCP geodetic longitude.
SCP.HAE	SCP HAE (m).
LOOK	Integer based on Side of Track parameter. SideOfTrack = L ⇔ LOOK = +1 SideOfTrack = R ⇔ LOOK = -1

Input WGS-84 Height:

HAE_0	The surface height (m) above the WGS-84 reference ellipsoid for projection point SPP^{TGT}.

Computed Parameters:

$\mathbf{SPP}^{TGT}$	Surface Projection Point (SPP^{TGT}) position on the HAE_0 surface and along the R/Rdot contour. The point SPP^{TGT} location in ECF coordinates is ($SPP.X^{TGT}$, $SPP.Y^{TGT}$, $SPP.Z^{TGT}$) and in geodetic coordinates ($SPP.Lat^{TGT}$, $SPP.Lon^{TGT}$, HAE_0).

User Selectable Algorithm Parameters:

ΔHAE_{MAX}	The height threshold for convergence of iterative projection sequence. Require $\Delta HAE_{MAX} > 0$. Recommended value: $\Delta HAE_{MAX} = 1.0$ m.
NLIM	Maximum number of iterations allowed. Recommended value: NLIM = 3.

A set of N ground planes that are tangent to the HAE_0 surface is computed. The ground planes are indexed by n = 1, 2, . . . , N. Ground Plane n is specified by a reference point in the plane, $GREF_n$, and the unit normal to the plane, $\mathbf{uGPN}_n$. Reference point $GREF_n$ is located on the HAE_0 surface. Ground plane n is also defined to be the geodetic ground plane at point $GREF_n$. See Section 5.1. Shown in Figure 9-2 is an example projection sequence where surface point SPP^{TGT} is computed after projecting to N = 2 ground planes. Shown in the figure are $GREF_1$, $\mathbf{uGPN}_1$ and initial projection point GPP_1. Also shown is Ground Plane 2 reference point $GREF_2$ located on the HAE_0 surface below the projection point GPP_1.

All position vectors are in ECF coordinates. All geodetic coordinates are in WGS-84 (Lat, Lon, HAE).

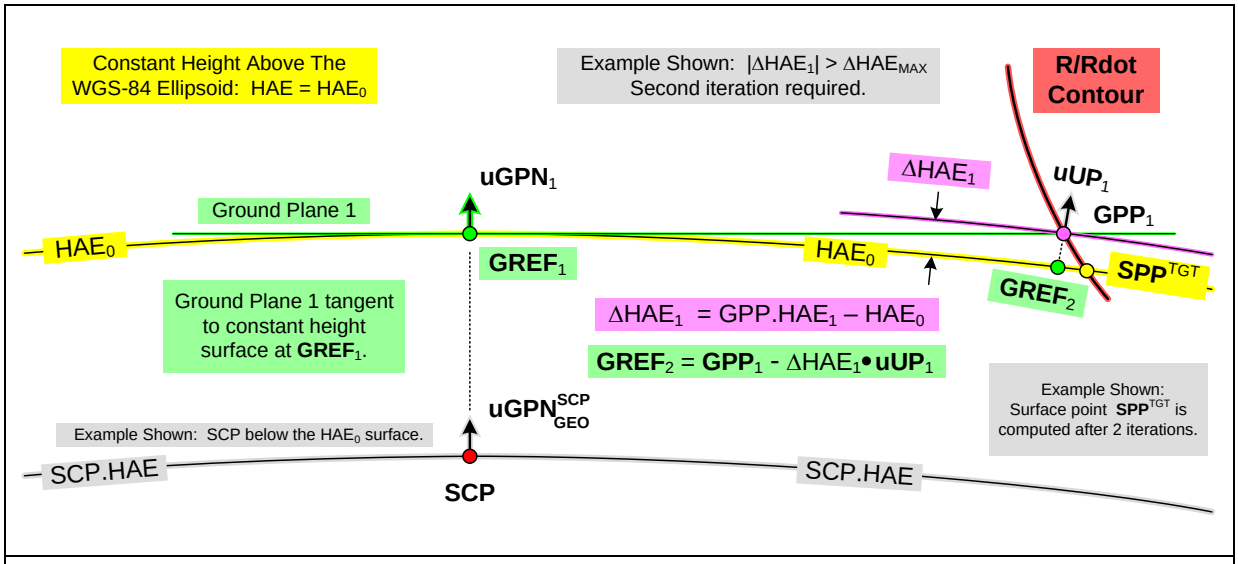


Figure 9-2 Initial Projection To Ground Plane 1

Example initial projection along R/Rdot yields position GPP_1. $GREF_2$ is computed from GPP_1.

R/Rdot Contour To Surface Point SPP^{TGT}

(1) Compute the geodetic ground plane normal at the SCP. Compute the parameters for Ground Plane 1. The reference point position is $\mathbf{GREF}_1$ and the unit normal is $\mathbf{uGPN}_1$.

$$\mathbf{uGPN}_1 = \mathbf{uGPN}_{\text{GEO}}^{\text{SCP}} = \begin{bmatrix} \cos(\text{SCP.Lat})\bullet\cos(\text{SCP.Lon}) \\ \cos(\text{SCP.Lat})\bullet\sin(\text{SCP.Lon}) \\ \sin(\text{SCP.Lat}) \end{bmatrix}$$

$$\mathbf{GREF}_1 = \mathbf{SCP} + \left(\text{HAE}_0 - \text{SCP.HAE}\right)\bullet\mathbf{uGPN}_{\text{GEO}}^{\text{SCP}}$$

Initialize the iteration count to n = 1. Iterate over the steps 2 through 4 below.

(2) Compute the precise projection along the R/Rdot contour to Ground Plane n. See Section 5. The result is ground plane point position $\mathbf{GPP}_n$. Convert from ECF coordinates to geodetic coordinates.

$$\boxed{\begin{array}{c}\text{Project}\\ \text{Along R/Rdot}\\ \text{To GP n}\end{array}} \rightarrow \mathbf{GPP}_n = \begin{bmatrix} GPP.X_n \\ GPP.Y_n \\ GPP.Z_n \end{bmatrix} \rightarrow \boxed{\begin{array}{c}\text{Convert}\\ \text{To}\\ \text{Geodetic LLH}\end{array}} \rightarrow \begin{bmatrix} GPP.Lat_n \\ GPP.Lon_n \\ GPP.HAE_n \end{bmatrix}$$

(3) Compute the unit vector in the increasing height direction at point $\mathbf{GPP}_n$, $\mathbf{uUP}_n$. Also compute the height difference at point $\mathbf{GPP}_n$ relative to the surface height HAE_0, ΔHAE_n.

$$\mathbf{uUP}_n = \begin{bmatrix} \cos(GPP.Lat_n)\bullet\cos(GPP.Lon_n) \\ \cos(GPP.Lat_n)\bullet\sin(GPP.Lon_n) \\ \sin(GPP.Lat_n) \end{bmatrix} \qquad \Delta HAE_n = GPP.HAE_n - HAE_0$$

(4) Test for the point $\mathbf{GPP}_n$ sufficiently close the HAE_0 surface ($\Delta HAE_n \leq \Delta HAE_{MAX}$) and for the maximum number of iterations to be reached (n = NLIM). For either condition, set final index value N = n, save parameters $\mathbf{GPP}_n$, $\mathbf{uUP}_n$ and ΔHAE_n and continue with Steps 5, 6 and 7. Otherwise, compute $\mathbf{GREF}_{n+1}$ and $\mathbf{uGPN}_{n+1}$, increment index n and repeat Steps 2, 3 and 4.

IF { $\Delta HAE_n \leq \Delta HAE_{MAX}$ OR n = NLIM }:

$\mathbf{GPP}_N = \mathbf{GPP}_n$ Final iteration index value N = n.

$\mathbf{uUP}_N = \mathbf{uUP}_n$ Save values for $\mathbf{GPP}_n$, $\mathbf{uUP}_n$ and ΔHAE_n

$\Delta HAE_N = \Delta HAE_n$ Continue with Steps 5, 6 and 7.

ELSE { $\Delta HAE_n > \Delta HAE_{MAX}$ AND n < NLIM }:

$\mathbf{GREF}_{n+1} = \mathbf{GPP}_n - \Delta HAE_n \bullet \mathbf{uUP}_n$ Compute $\mathbf{GREF}_{n+1}$ on the HAE_0 surface.

$\mathbf{uGPN}_{n+1} = \mathbf{uUP}_n$

Increment index n: $n \leftarrow n+1$. Repeat Steps 2, 3 and 4.

(5) Compute the unit slant plane normal vector, **uSPN**, that is tangent to the R/Rdot contour at point $\mathbf{GPP}_N$. Unit vector **uSPN** points away from the center of the earth and in a direction of increasing HAE at $\mathbf{GPP}_N$.

$$\mathbf{SPN} = \text{LOOK}\bullet\mathbf{VARP}_{COA}^{TGT} \times (\mathbf{GPP}_N - \mathbf{ARP}_{COA}^{TGT}) \qquad \mathbf{uSPN} = \frac{1}{|\mathbf{SPN}|}\bullet\mathbf{SPN}$$

(6) Compute the straight line projection from point $\mathbf{GPP}_N$ along the slant plane normal to point SLP. See Figure 9.3. Point SLP is located is very close to the precise R/Rdot contour

intersection with the constant height surface. Convert the position of point SLP from ECF coordinates to geodetic coordinates.

$$\mathrm{SF} = \mathbf{uUP}_{\mathrm{N}} \bullet \mathbf{uSPN} \qquad \mathbf{SLP} = \mathbf{GPP}_{\mathrm{N}} - \left(\Delta\mathrm{HAE}_{\mathrm{N}} \bullet \frac{1}{\mathrm{SF}}\right) \bullet \mathbf{uSPN}$$

$$\mathbf{SLP} = \begin{bmatrix} \mathrm{SLP.X} \\ \mathrm{SLP.Y} \\ \mathrm{SLP.Z} \end{bmatrix} \rightarrow \boxed{\begin{matrix} \text{Convert} \\ \text{To} \\ \text{Geodetic LLH} \end{matrix}} \rightarrow \begin{bmatrix} \mathrm{SLP.Lat} \\ \mathrm{SLP.Lon} \\ \mathrm{SLP.HAE} \end{bmatrix}$$

(7) Assign surface point $\mathrm{SPP}^{\mathrm{TGT}}$ position as follows by adjusting the HAE to be on the HAE_0 surface. Convert from geodetic coordinates to ECF coordinates.

$$\begin{aligned} \mathrm{SPP.Lat}^{\mathrm{TGT}} &= \mathrm{SLP.Lat} \\ \mathrm{SPP.Lon}^{\mathrm{TGT}} &= \mathrm{SLP.Lon} \\ \mathrm{SPP.HAE}^{\mathrm{TGT}} &= \mathrm{HAE}_0 \end{aligned} \qquad \begin{bmatrix} \mathrm{SPP.Lat}^{\mathrm{TGT}} \\ \mathrm{SPP.Lon}^{\mathrm{TGT}} \\ \mathrm{SPP.HAE}^{\mathrm{TGT}} \end{bmatrix} \rightarrow \boxed{\begin{matrix} \text{Convert} \\ \text{To} \\ \text{ECF XYZ} \end{matrix}} \rightarrow \begin{bmatrix} \mathrm{SPP.X}^{\mathrm{TGT}} \\ \mathrm{SPP.Y}^{\mathrm{TGT}} \\ \mathrm{SPP.Z}^{\mathrm{TGT}} \end{bmatrix}$$

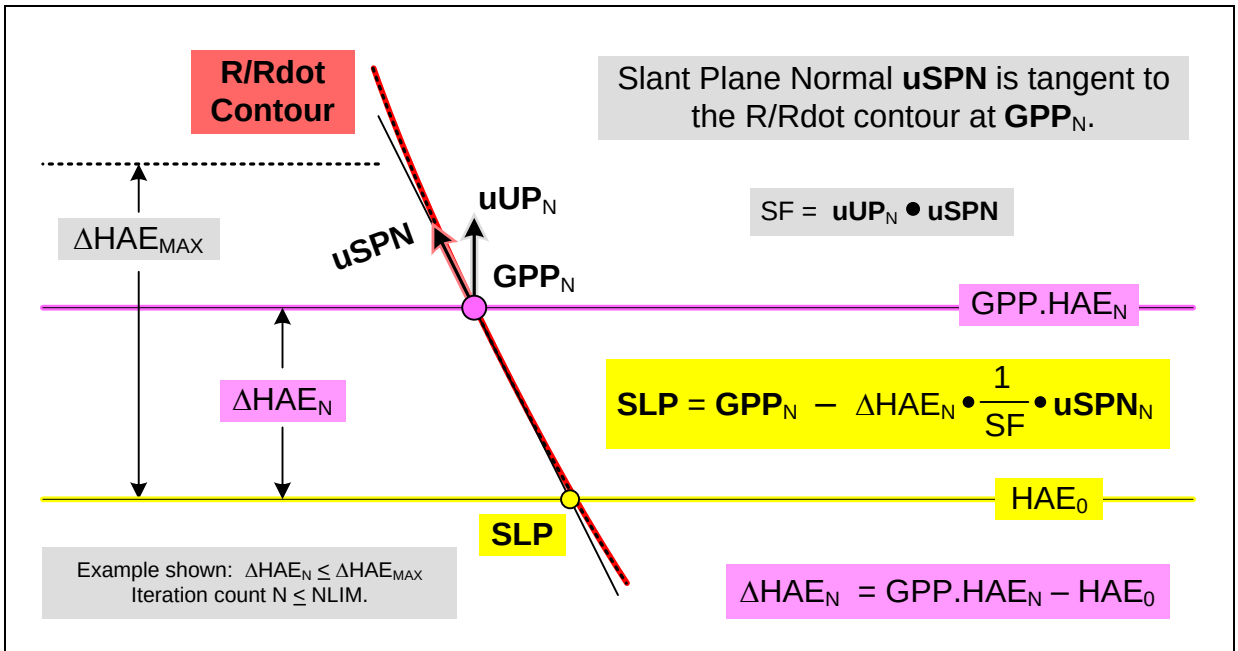


Figure 9-3 Final Straight Line Projection
Projected point SLP is very close to the constant height surface.

9.3 Projection Algorithm Comments

The projection algorithm has been developed with consideration for both accuracy and computational efficiency. The following comments are provided for clarification.

(1) For most projection computations, the number of ground plane projections computed will be either 1 or 2. For the suggested value of ΔHAE_{MAX} = 1.0 m, the iteration stopping criteria is met when the projection point GPP_n is displaced less than 3.5 km from the reference point $GREF_n$. Even for large area images, the maximum iteration count expected is N = 2.

(2) The suggested height threshold of ΔHAE_{MAX} = 1.0 m will yield highly accurate projection results for all SAR imaging geometries. The straight line projection defined in Step 6 and shown in Figure 9.3 is highly accurate even for very short range and steep grazing angle geometries. Reducing ΔHAE_{MAX} will have negligible improvement in projection accuracy.

(3) Iteration count limit NLIM is included for safety or for the rare user that chooses a value for ΔHAE_{MAX} less than 1.0 m.

(4) For the unexpected condition that the R/Rdot contour does not intersect the HAE_0 surface, the condition can be detected by the failure of the initial R/Rdot projection to Ground Plane 1. See Section 5 for this condition. Including the check after each ground plane projection may also be included. Such a condition is most likely due to an error in computing the projection parameters or an error in choosing surface height HAE_0.

10 Precise R/Rdot To DEM Surface Projection

The precise projection to an earth surface model along an R/Rdot contour is described. The R/Rdot contour is relative to an ARP Center Of Aperture position and velocity. The earth surface is described by a Digital Elevation Model (DEM) that defines a unique surface height as a function of two horizontal coordinates. The projection computation may yield one or more surface points that lie along the R/Rdot contour.

10.1 Surface Model Description

A typical DEM surface is defined by a two-dimensional array of height values, denoted {HD(m,n)}. The height values are the surface heights relative to a reference surface. The DEM reference surface is spanned by two horizontal coordinates, denoted XD and YD. Surface heights {HD(m,n)} are the heights at surface points with horizontal coordinates {XD(m)} and {YD(n)}. The reference surface, also the surface of height HD = 0, is approximately parallel to a surface of constant WGS-84 HAE for the region of the earth spanned by the DEM surface. For any position with horizontal coordinates (XD, YD), a unique surface height HD(XD,YD) may be computed by an appropriate interpolation of the height array values.

An example DEM surface is shown in Figure 10-1. The surface is defined by an array of surface points of size MD x ND. The left-hand portion of the figure shows the surface relative to the earth tangent plane defined at the SCP. Shown is an East, North, Up (ENU) coordinate frame with origin at the SCP where the Up direction is normal to the ground plane. The right-hand portion of the figure shows the region of the DEM reference surface spanned by the two-dimensional array of locations {(XD(m),YD(n), 0), m = 0 to MD-1, n = 0 to ND-1}. The locations on the DEM reference surface projected onto the ground plane are an array of approximately equally spaced points.

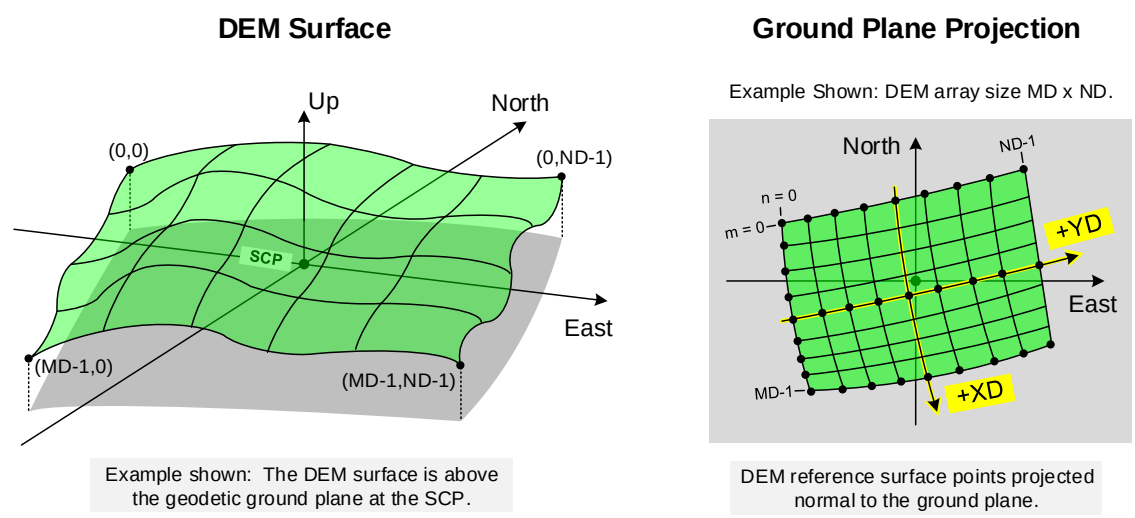


Figure 10-1 Example DEM Surface
Surface defined by two-dimensional height array {HD(m,n)}of size MD x ND points.

The coordinate systems used in commonly available DEM data sets include the following examples. Both systems meet the general DEM coordinate system described above.

(1) The horizontal coordinates (XD, YD) are geodetic latitude and longitude. The height values {HD(m,n)} are Height Above Mean Sea Level (HMSL). The height samples are evenly spaced along lines of constant latitude and constant longitude.

(2) The horizontal coordinates (XD, YD) are UTM coordinates Easting and Northing. The height values {HD(m,n)} are either WGS-84 HAE or HMSL. The height samples are evenly spaced along lines of constant Easting and constant Northing for a given UTM zone.

The projection algorithm described below assumes the surface model meets the following set of modest constraints. Most available DEM surface data sets meet these constraints. Let point S be any point on the DEM surface. Point S is located in DEM coordinates at (S.XD, S.YD, S.HD) and in geodetic coordinates at (S.Lat, S.Lon, S.HAE).

(1) For the contour of points with constant horizontal coordinates (S.XD, S.YD), there is one and only one surface point (at height S.HD).

(2) For the contour of points with constant horizontal coordinates (S.XD, S.YD), moving in the increasing HD direction is always in the increasing HAE direction.

(3) For the line of points with constant latitude and longitude (S.Lat, S.Lon), the line intersects the surface at one and only one point (at height S.HAE).

(4) For the line of points with constant latitude and longitude (S.Lat, S.Lon), moving in the increasing HAE direction is always in the increasing HD direction.

The projection algorithm includes the following generic functions that are dependent upon DEM coordinate system. These functions are well defined given the DEM coordinate system. Let point P be any point in the three-dimensional scene. Point P is located in ECF coordinates at (P.X, P.Y, P.Z) and in DEM coordinates at (P.XD, P.YD, P.HD).

(1) Conversion From ECF to DEM coordinates. An input position in ECF coordinates with components expressed in meters is converted to DEM coordinates. Height coordinate P.HD is in meters.

$$\mathbf{P}_{\mathrm{ECF}} = \begin{bmatrix} \mathrm{P.X} \\ \mathrm{P.Y} \\ \mathrm{P.Z} \end{bmatrix} \rightarrow \boxed{\begin{matrix} \text{Convert ECF} \\ \text{To} \\ \text{DEM Coords} \end{matrix}} \rightarrow \mathbf{P}_{\mathrm{DEM}} = \begin{bmatrix} \mathrm{P.XD} \\ \mathrm{P.YD} \\ \mathrm{P.HD} \end{bmatrix}$$

(2) Get DEM Height At (XD, YD). Determine the DEM surface height HD from DEM horizontal coordinates (XD, YD). Height value HD is in meters.

$$(XD, YD) \rightarrow \boxed{\begin{matrix}\text{Get Surface}\\ \text{Height HD}\end{matrix}} \rightarrow HD \rightarrow \begin{bmatrix} XD \\ YD \\ HD \end{bmatrix}$$

Surface point in DEM Coordinates

10.2 R/Rdot Contour Angle, CA

The R/Rdot projection contour is a circle located in a plane normal to the ARP velocity vector at COA. See Figure 10-2. The center point, denoted **CTR**, is located along the line defined by the ARP position and the ARP velocity vector. The radius of the circle, denoted R_{RRC}, is computed from the R_{COA} and the Doppler Cone Angle (DCA) to the contour. A pair of orthogonal unit vectors that lie in the plane, denoted **uRRX** and **uRRY**, are computed. Unit vector **uRRX** points toward the ground track. Unit vector **uRRY** points to the left side of the ground track. Points along the contour may be defined by the Contour Angle, denoted CA. The location of a point **P** on the contour at contour angle CA is computed as shown below. For points on the left side of the ground track, the contour angle CA is positive. For points on the right side of the ground track, the contour angle is negative.

$$\mathbf{P} = \mathbf{CTR} + R_{RRC} \bullet \left(\cos(CA) \bullet \mathbf{uRRX} + \sin(CA) \bullet \mathbf{uRRY} \right)$$

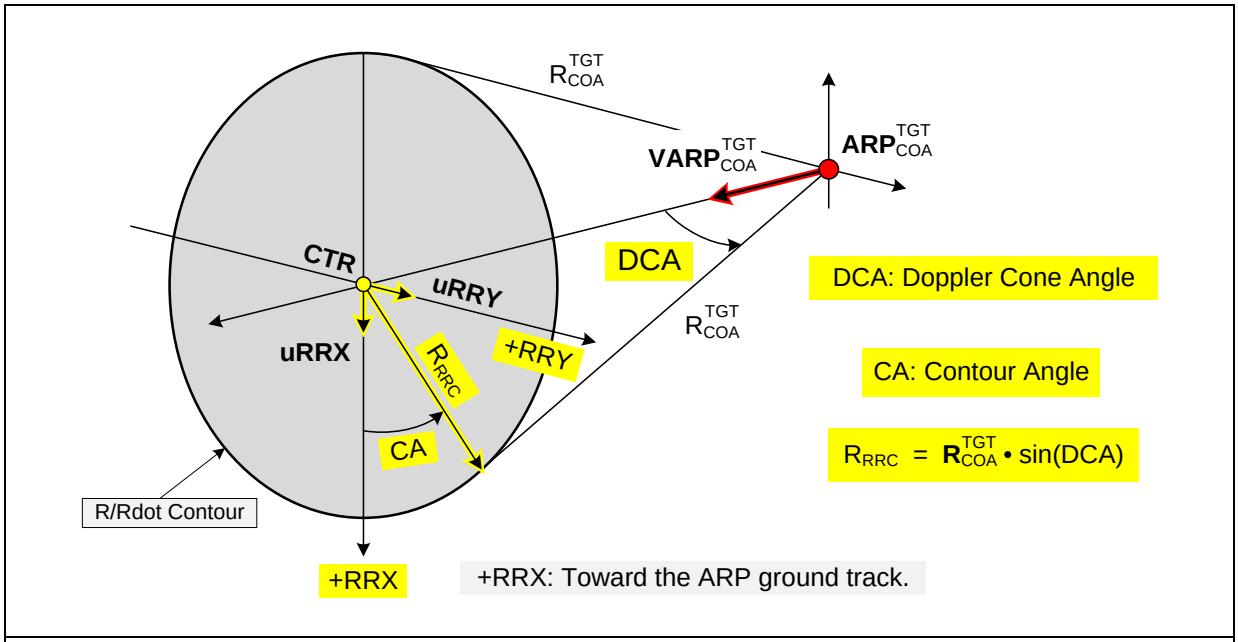


Figure 10-2 Projection Contour & Contour Angle
Points along the R/Rdot contour are on a circle centered at point CTR.

The approach used to determine the intersection points of the R/Rdot contour and the DEM surface is as follows. Refer to Figure 10-3. For the given DEM surface, the minimum and maximum surface heights relative to the WGS-84 reference ellipsoid are determined. The heights are HAE_{MIN} and HAE_{MAX}. The R/Rdot contour intersects the surface of constant HAE = HAE_{MAX} at point A and the surface of constant HAE = HAE_{MIN} at point B. The intersection point(s) of the R/Rdot contour and the DEM surface are computed by searching the contour between points A and B. A set of NPTS points on the R/Rdot contour from point A to point B are computed. Each contour point position is converted to DEM coordinates. The height of each contour point is then compared to the DEM surface height. The contour and DEM surface intersection points are determined by finding contour points "on" the surface (i.e. within a tolerance) or where adjacent contour points that are off the surface but with an intersection point between them (i.e. one point below the surface and the adjacent point above the surface).

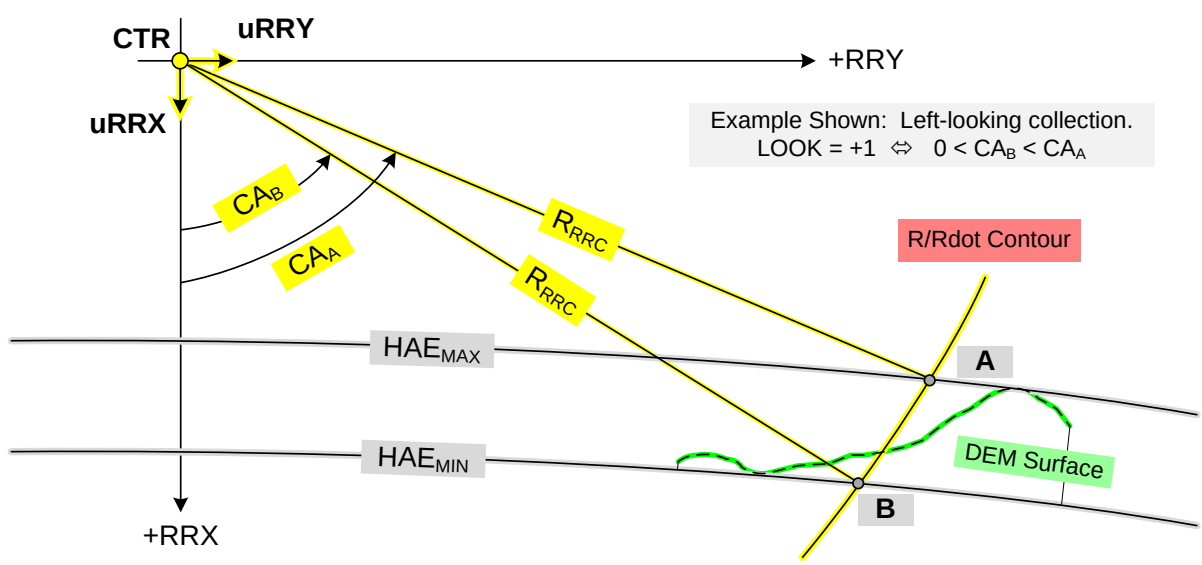


Figure 10-3 Projection To Bounding Surfaces
All DEM surface points lie between heights HAE_{MIN} and HAE_{MAX}.

The algorithm makes the following assumptions about the DEM surface and the R/Rdot contour computed for the selected pixel location in the image. See Figure 10-4.

(1) The R/Rdot contour intersects the constant HAE surfaces at HAE_{MIN} and HAE_{MAX}. See Section 9 for the projection to a surface of constant HAE. Point B is always on or below the DEM surface.

(2) The set of NPTS contour points are denoted $\{\mathbf{P}_n\}$, indexed n = 1 to NPTS. First point $\mathbf{P}_1$ is coincident with point B. Last point $\mathbf{P}_{NPTS}$ is above point A and is always above the DEM surface.

(3) The R/Rdot contour intersects the DEM surface at a minimum of one point. The set of MPTS surface points is denoted $\{\mathbf{S}_m\}$, indexed m = 1 to MPTS. Figure 10-4 shows an example with MPTS = 3.

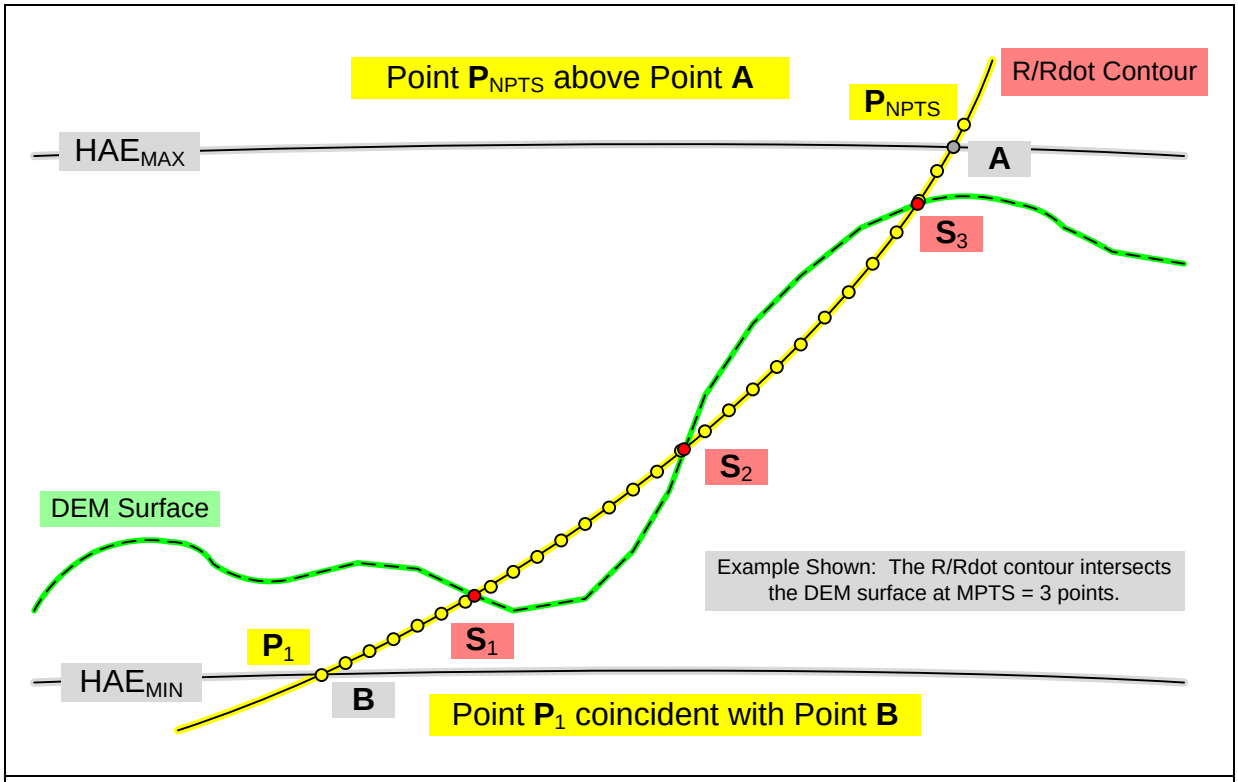


Figure 10-4 DEM Surface Projection Points
Example showing multiple R/Rdot contour and DEM surface intersections.

10.3 R/Rdot Contour To DEM Surface Intersection

The projection of image pixel grid location ($xrow^{TGT}$, $ycol^{TGT}$) to a DEM surface along an R/Rdot contour is described below. The input parameters include the COA projection set that defines the R/Rdot contour, the image SCP parameters (used to project to the minimum and maximum HAE surfaces) and the DEM surface parameters. The algorithm computes one or more surface points $\{\mathbf{S}_m\}$for which the R/Rdot contour intersects the DEM surface.

Image Pixel Grid Location COA Projection Set:

$\mathbf{ARP}_{COA}^{TGT}$	ARP position at the COA time in ECF coordinates. See Section 2.
$\mathbf{VARP}_{COA}^{TGT}$	ARP velocity at the COA time in ECF coordinates.
R_{COA}^{TGT}	Range of the R/Rdot contour. See Section 4.
$Rdot_{COA}^{TGT}$	Range rate (dR/dt) of the R/Rdot contour.

Image Parameters:

SCP	SCP position in ECF coordinates.
SCP.Lat	SCP geodetic latitude.
SCP.Lon	SCP geodetic longitude.
SCP.HAE	SCP HAE (m).
LOOK	Integer based on Side of Track parameter. SideOfTrack = L ⇔ LOOK = +1 SideOfTrack = R ⇔ LOOK = -1

DEM Surface Parameters:

{HD(m, n)}	DEM surface height array values (m).
{XD(m), YD(n)}	DEM horizontal coordinate sample locations.
HAE_{MIN}	Minimum WGS-84 HAE for any point on the DEM surface (m). Note: Any HAE value below all surface points may be used.
HAE_{MAX}	Maximum WGS-84 HAE for any point on the DEM surface (m). Note: Any HAE value above all surface points may be used.
$\Delta DIST_{DEM}$	Maximum horizontal distance between surface points for which the surface is well approximated by a straight line between the points (m). Recommended value: $\Delta DIST_{DEM}$ = 0.5 x minimum DEM horizontal sample spacing (m). The horizontal spacing is typically referred to as the "post spacing".

Computed Parameters:

MPTS	Number of projected point(s) on the DEM surface. MPTS $\geq$ 1.
$\{\mathbf{S}_m\}$	Set of point(s) where the R/Rdot contour intersects the DEM surface. Points are indexed m = 1 to MPTS. The point $\mathbf{S}_m$ in located in ECF coordinates at ($S.X_m$, $S.Y_m$, $S.Z_m$) and in geodetic coordinates at ($S.Lat_m$, $S.Lon_m$, $S.HAE_m$).

User Selectable Algorithm Parameters:

$\Delta DIST_{RRC}$	Maximum distance between adjacent points along the R/Rdot contour. Require $\Delta DIST_{RRC} > 0$. Recommended value: $\Delta DIST_{RRC}$ = 10.0 m.
ΔHD_{LIM}	Height difference threshold for determining if a point on the R/Rdot contour is on the DEM surface (m). Recommended value: ΔHD_{LIM} = 0.001 m.

R/Rdot Contour To DEM Surface Projection

(1) Compute the center point, **CTR**, and the radius of the R/Rdot projection contour, R_{RRC}.

$$\mathrm{VMAG} = \left|\mathbf{VARP}_{\mathrm{COA}}^{\mathrm{TGT}}\right| \qquad \mathbf{uVEL} = \frac{1}{\mathrm{VMAG}} \bullet \mathbf{VARP}_{\mathrm{COA}}^{\mathrm{TGT}}$$

$$\mathrm{cos_DCA} = -\frac{1}{\mathrm{VMAG}} \bullet \mathrm{Rdot}_{\mathrm{COA}}^{\mathrm{TGT}} \qquad \mathrm{sin_DCA} = +\left(1 - \mathrm{cos_DCA}^2\right)^{0.5}$$

$$\mathbf{CTR} = \mathbf{ARP}_{\mathrm{COA}}^{\mathrm{TGT}} + \mathrm{R}_{\mathrm{COA}}^{\mathrm{TGT}} \bullet \mathrm{cos_DCA} \bullet \mathbf{uVEL}$$

$$\mathrm{R}_{\mathrm{RRC}} = \mathrm{R}_{\mathrm{COA}}^{\mathrm{TGT}} \bullet \mathrm{sin_DCA}$$

(2) Compute the unit vectors **uRRX** and **uRRY** to be used to compute points located on the R/Rdot contour.

$$\mathrm{DEC_ARP} = \left|\mathbf{ARP}_{\mathrm{COA}}^{\mathrm{TGT}}\right| \qquad \mathbf{uUP} = \frac{1}{\mathrm{DEC_ARP}} \bullet \mathbf{ARP}_{\mathrm{COA}}^{\mathrm{TGT}}$$

$$\mathbf{RRY} = \underbrace{\mathbf{uUP} \times \mathbf{uVEL}}_{\text{Vector Cross Product}} \qquad \mathbf{uRRY} = \frac{1}{\left|\mathbf{RRY}\right|} \bullet \mathbf{RRY}$$

$$\mathbf{uRRX} = \underbrace{\mathbf{uRRY} \times \mathbf{uVEL}}_{\text{Vector Cross Product}}$$

(3) Compute the projection along the R/Rdot contour to the surface of constant HAE at height HAE_{MAX}. The projection point at height HAE_{MAX} is point **A**. See Section 9. Also compute the cosine and sine of the contour angle to point **A**, cos_CA_A and sin_CA_A.

$$\left\{\mathbf{ARP}_{COA}^{TGT} \quad \mathbf{VARP}_{COA}^{TGT} \quad R_{COA}^{TGT} \quad Rdot_{COA}^{TGT}\right\} \rightarrow \begin{array}{c}\text{Project To}\\ \text{Surface Of}\\ \text{Constant HAE}\end{array} \rightarrow \mathbf{A} = \begin{bmatrix} A.X \\ A.Y \\ A.Z \end{bmatrix}$$

$$HAE_{MAX} \rightarrow$$

$$cos_CA_A = \frac{1}{R_{RRC}} \bullet \underbrace{\left(\mathbf{A} - \mathbf{CTR}\right) \bullet \mathbf{uRRX}}_{\text{Vector Dot Product}} \qquad sin_CA_A = LOOK \bullet \left(1 - (cos_CA_A)^2\right)^{0.5}$$

(4) Compute the projection along the R/Rdot contour to the surface of constant HAE at height HAE_{MIN}. The projection point at height HAE_{MIN} is point **B**. Also compute the cosine and sine of the contour angle to point **B**, cos_CA_B and sin_CA_B.

$$\left\{\mathbf{ARP}_{COA}^{TGT} \quad \mathbf{VARP}_{COA}^{TGT} \quad R_{COA}^{TGT} \quad Rdot_{COA}^{TGT}\right\} \rightarrow \begin{array}{c}\text{Project To}\\ \text{Surface Of}\\ \text{Constant HAE}\end{array} \rightarrow \mathbf{B} = \begin{bmatrix} B.X \\ B.Y \\ B.Z \end{bmatrix}$$

$$HAE_{MIN} \rightarrow$$

$$cos_CA_B = \frac{1}{R_{RRC}} \bullet \underbrace{\left(\mathbf{B} - \mathbf{CTR}\right) \bullet \mathbf{uRRX}}_{\text{Vector Dot Product}} \qquad sin_CA_B = LOOK \bullet \left(1 - (cos_CA_B)^2\right)^{0.5}$$

(5) A set of NPTS points along the R/Rdot contour are to be computed. The points will be spaced in equal increments of the cosine of the contour angle. Compute the step size, Δcos_CA, as follows.

(1) Step size Δcos_RRC is computed such that the distance between adjacent points on the R/Rdot contour is approximately equal to $\Delta DIST_{RRC}$.

$$\Delta cos_RRC = \Delta DIST_{RRC} \bullet \frac{1}{R_{RRC}} \bullet \left|sin_CA_B\right|$$

(2) Step size Δcos_DEM is computed such that the horizontal distance between adjacent points on the R/Rdot contour is approximately equal to $\Delta DIST_{DEM}$.

$$\Delta cos_DEM = \Delta DIST_{DEM} \bullet \frac{1}{R_{RRC}} \bullet \frac{\left|sin_CA_B\right|}{cos_CA_B}$$

(3) Set Δcos_CA as follows. Note the value of $\Delta cos_CA < 0$.

$$\Delta cos_CA = -MIN\left(\Delta cos_RRC, \quad \Delta cos_DEM\right)$$

(6) Determine the number of points along the R/Rdot contour to be computed, NPTS.

$$\mathrm{NPTS} = \mathrm{FLOOR}\left(\frac{\mathrm{cos_CA_A} - \mathrm{cos_CA_B}}{\Delta\mathrm{cos_CA}}\right) + 2 \qquad \text{Note: } \mathrm{cos_CA_A} - \mathrm{cos_CA_B} \le 0$$

(7) Compute the set of NPTS along the R/Rdot contour, $\{\mathbf{P}_n\}$ for n =1, 2, . . . , NPTS. Initial point $\mathbf{P}_1$ is located on the $\mathrm{HAE_{MIN}}$ surface. Final point $\mathbf{P}_{\mathrm{NPTS}}$ is located above the $\mathrm{HAE_{MAX}}$ surface. Point $\mathbf{P}_n$ is computed in ECF coordinates as follows.

For n = 1, 2, . . . , NPTS:

$$\mathrm{cos_CA_n} = \mathrm{cos_CA_B} + (n-1)\bullet\Delta\mathrm{cos_CA}$$

$$\mathrm{sin_CA_n} = \mathrm{LOOK}\bullet\left(1-(\mathrm{cos_CA_n})^2\right)^{0.5}$$

$$\mathbf{P}_n = \mathbf{CTR} + \mathrm{R_{RRC}}\bullet\left(\mathrm{cos_CA_n}\bullet\mathbf{uRRX} + \mathrm{sin_CA_n}\bullet\mathbf{uRRY}\right)$$

(8) For each of the NPTS points, convert from ECF coordinates to DEM coordinates. Also compute the DEM surface height for the point with DEM horizontal coordinates ($\mathrm{P.XD_n}$, $\mathrm{P.YD_n}$), $\mathrm{HDEM_n}$. Height coordinates $\mathrm{P.HD_n}$ and $\mathrm{HDEM_n}$ are in meters.

For n = 1, 2, . . . , NPTS:

$$\mathbf{P}_n = \underbrace{\begin{bmatrix} \mathrm{P.X_n} \\ \mathrm{P.Y_n} \\ \mathrm{P.Z_n} \end{bmatrix}}_{\substack{\text{ECF}\\ \text{Coordinates}}} \rightarrow \boxed{\begin{matrix}\text{Convert ECF}\\ \text{To}\\ \text{DEM Coords}\end{matrix}} \rightarrow \mathbf{P}_n = \underbrace{\begin{bmatrix} \mathrm{P.XD_n} \\ \mathrm{P.YD_n} \\ \mathrm{P.HD_n} \end{bmatrix}}_{\substack{\text{DEM}\\ \text{Coordinates}}}$$

$$(\mathrm{P.XD_n}, \mathrm{P.YD_n}) \rightarrow \boxed{\begin{matrix}\text{Get Surface}\\ \text{Height HD}\end{matrix}} \rightarrow \mathrm{HDEM_n} \rightarrow \underbrace{\begin{bmatrix} \mathrm{P.XD_n} \\ \mathrm{P.YD_n} \\ \mathrm{HDEM_n} \end{bmatrix}}_{\substack{\text{Surface point in}\\ \text{DEM Coordinates}}}$$

(9) For each of the NPTS points, compute the difference in height, $\Delta\mathrm{HD_n}$, between the point on the contour and DEM surface point. Also, set the parameter $\mathrm{AOB_n}$ to indicate that point $\mathbf{P}_n$ is ABOVE, ON, or BELOW the DEM surface. Contour points that are within $\Delta\mathrm{HD_{LIM}}$ of the surface point are considered to be “on” the surface.

For n = 1, 2, . . . , NPTS:

$$\Delta HD_n = P.HD_n - HDEM_n$$

$$AOB_n = \begin{cases} +1, & \text{If } \Delta HD_n > \Delta HD_{LIM} \\ 0, & \text{Else if } |\Delta HD_n| \le \Delta HD_{LIM} \\ -1, & \text{Otherwise} \end{cases}$$

(10) Determine the set of MPTS points where the R/Rdot contour intersects the DEM surface, $\{\mathbf{S}_m\}$. Points with indicator $AOB_n = 0$ are accepted as being on the surface. A surface point is computed between points P_n and P_{n+1} when both are "off" the surface and the R/Rdot contour intersects the surface between them (i.e. AOB_n x AOB_{n+1} = -1). Surface points are indexed m = 1, 2, . . ., MPTS.

Initialize the count to zero: m = 0

Loop over n = 1, 2, . . . , NPTS – 1:

If $AOB_n = 0$: Point $\mathbf{P}_n$ is acceptably close to the surface.

$$m \leftarrow m+1 \qquad \mathbf{S}_m = \mathbf{P}_n$$

If $AOB_n \bullet AOB_{n+1} = -1$: Compute the surface point between $\mathbf{P}_n$ and $\mathbf{P}_{n+1}$.

$$m \leftarrow m+1 \qquad frac = \frac{\Delta HD_n}{\Delta HD_n - \Delta HD_{n+1}}$$

$$cos_CA_S = cos_CA_B + (n - 1 + frac) \bullet \Delta cos_CA$$

$$sin_CA_S = LOOK \bullet \left(1 - (cos_CA_S)^2\right)^{0.5}$$

$$\mathbf{S}_m = \mathbf{CTR} + R_{RRC} \bullet (cos_CA_S \bullet \mathbf{uRRX} + sin_CA_S \bullet \mathbf{uRRY})$$

End loop. For n < NPTS – 1: Next n.

Set MPTS = Number of surface points found. Surface points $\{\mathbf{S}_m\}$will be in the order of increasing WGS-84 HAE.

11 Projection Sensitivity Parameters

Image grid location **IL** is a point in the two-dimensional image grid. Scene point **PT** is a point in the imaged scene that lies along the R/Rdot projection contour for image location **IL**. Given scene point position **PT**, image location **IL** may be computed using the Scene To Image projection function. Also, given image location **IL** and a surface model for which scene point **PT** is a point on the surface, scene point **PT** may be computed using the Image To Scene projection function. Image location **IL** and scene point **PT** are referred to as an image location/scene point projection pair or, simply, an image/scene projection pair.

The image projection functions that connect the image projection pair **IL** and **PT** are shown in Figure 11-1. The projection functions make use of the image metadata and a surface model for the imaged scene. The ARP position versus time and the computed range may be adjusted via the Adjustable Parameter Offsets. See Section 8. The unit normal to the scene surface model at point **PT** is **uGPN**. For a smoothly varying surface model, the surface near **PT** may be approximated by the ground plane defined by point **PT** and normal vector **uGPN**.

$$\text{Image Location: } \mathbf{IL} = \begin{bmatrix} \text{xrow} \\ \text{ycol} \end{bmatrix} \qquad \text{Scene Point (ECF): } \mathbf{PT} = \begin{bmatrix} \text{PT_X} \\ \text{PT_Y} \\ \text{PT_Z} \end{bmatrix}$$

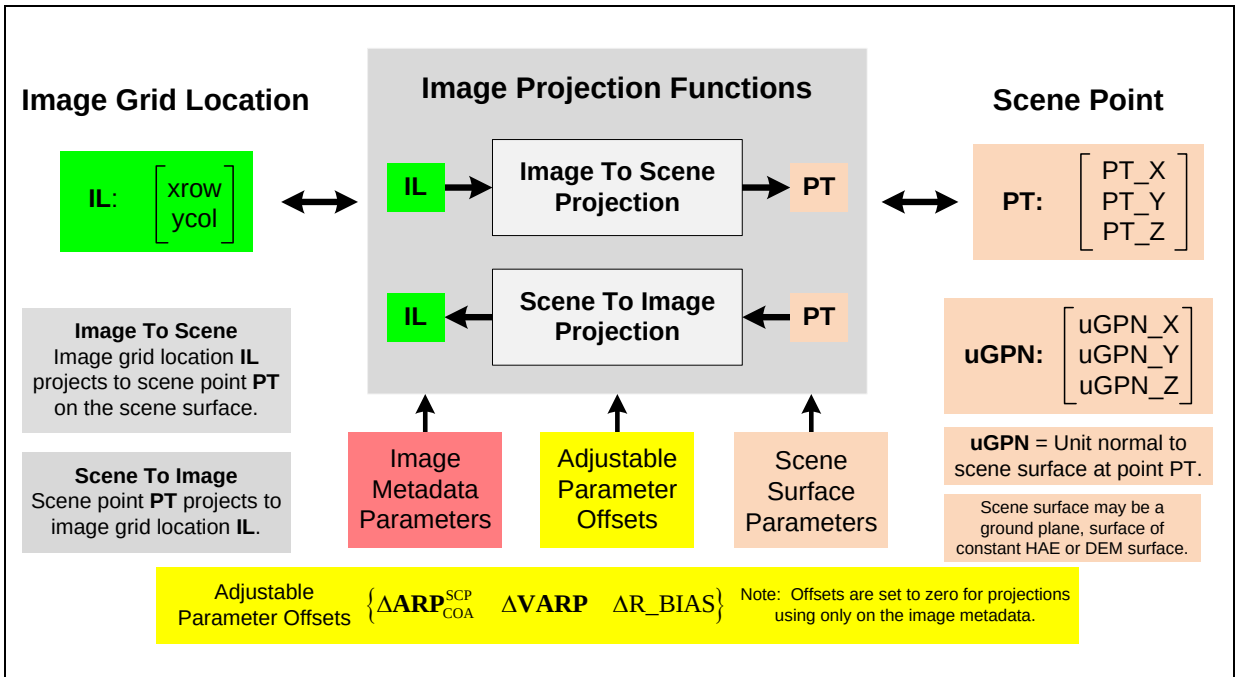


Figure 11-1 Image Location/Scene Point Projection Pair
Image grid location IL projects to scene point position PT on the scene surface model.

For a given image/scene projection pair, a set of projection sensitivity parameters may be computed. The sensitivity parameters relate how a change in one parameter will effect changes in another parameter given all other parameters are held constant. For example, let image location **IL1** and scene point **PT1** be an image projection pair. A change in scene point position, Δ**PT**, will cause a change in image location, Δ**IL**, such that the new image location and scene point are also an image/scene projection pair. Let the new scene point be **PT2** and the new image location be **IL2**. Scene points and image locations are related as follows.

$$\mathbf{PT1} \rightarrow \mathbf{PT2} = \mathbf{PT1} + \Delta\mathbf{PT} \quad \Rightarrow \quad \mathbf{IL1} \rightarrow \mathbf{IL2} = \mathbf{IL1} + \Delta\mathbf{IL}$$

where: $$\Delta\mathbf{PT} = \begin{bmatrix} \Delta\mathrm{PT_X} \\ \Delta\mathrm{PT_Y} \\ \Delta\mathrm{PT_Z} \end{bmatrix} \qquad \Delta\mathbf{IL} = \begin{bmatrix} \Delta\mathrm{xrow} \\ \Delta\mathrm{ycol} \end{bmatrix}$$

The sensitivity parameters are defined in matrix form. For the example above, the change in image location Δ**IL** may be estimated from the change in scene point Δ**PT** using the 2 x 3 sensitivity matrix $\mathbf{M}_{\mathrm{PT}}^{\mathrm{IL}}$. For small changes in scene point position Δ**PT**, the change in image location may be accurately computed.

$$\Delta\mathbf{IL} = \mathbf{M}_{\mathrm{PT}}^{\mathrm{IL}} \bullet \Delta\mathbf{PT} \qquad \mathbf{M}_{\mathrm{PT}}^{\mathrm{IL}} = \begin{bmatrix} \frac{\Delta\mathrm{xrow}}{\Delta\mathrm{PT_X}} & \frac{\Delta\mathrm{xrow}}{\Delta\mathrm{PT_Y}} & \frac{\Delta\mathrm{xrow}}{\Delta\mathrm{PT_Z}} \\ \frac{\Delta\mathrm{ycol}}{\Delta\mathrm{PT_X}} & \frac{\Delta\mathrm{ycol}}{\Delta\mathrm{PT_Y}} & \frac{\Delta\mathrm{ycol}}{\Delta\mathrm{PT_Z}} \end{bmatrix}$$

For a SAR image formed with either a fixed COA time or smoothly varying COA times, the shape of the R/Rdot projection contours vary smoothly across the imaged scene. In the example above, for image locations near **IL1** that project to scene points near **PT1**, the projection contours near **PT1** may be approximated by straight lines that are normal to the COA slant plane for scene point **PT1**. See Section 11.2. A given projection contour can be specified by its slant plane projection point (SPX, SPY). Hence, the slant plane projection point, denoted **SPXY**, may be used to relate scene points near **PT1** and image grid locations near **IL1**.

$$\Delta\mathbf{PT} = \begin{bmatrix} \Delta\mathrm{PT_X} \\ \Delta\mathrm{PT_Y} \\ \Delta\mathrm{PT_Z} \end{bmatrix} \quad \Leftrightarrow \quad \mathbf{SPXY} = \begin{bmatrix} \mathrm{SPX} \\ \mathrm{SPY} \end{bmatrix} \quad \Leftrightarrow \quad \Delta\mathbf{IL} = \begin{bmatrix} \Delta\mathrm{xrow} \\ \Delta\mathrm{ycol} \end{bmatrix}$$

In the example above, parameter Δ**PT** is referred to as the independent parameter and parameter Δ**IL** is referred to as the dependent parameter. For the sensitivity matrices to be computed, the independent parameters include the image grid location, the scene point

position and the Adjustable Parameters Offsets. The dependent parameters include the image grid location or the scene point position. The parameters are summarized Table 11-1 below.

Table 11-1 Projection Sensitivity Parameters	
$\Delta\mathbf{IL} = \begin{bmatrix} \Delta\text{xrow} \\ \Delta\text{ycol} \end{bmatrix}$	A change in image grid location specified in SCP-centered image coordinates (in meters).
$\Delta\mathbf{PT} = \begin{bmatrix} \Delta\text{PT_X} \\ \Delta\text{PT_Y} \\ \Delta\text{PT_Z} \end{bmatrix}$	A change in scene point position specified in ECF coordinates (in meters).
$\Delta\mathbf{GPXY} = \begin{bmatrix} \Delta\text{GPX} \\ \Delta\text{GPY} \end{bmatrix}$	A change in scene point position that is constrained to the ground plane in ground plane coordinates GPX and GPY (in meters). See Section 11.2.
$\Delta\mathbf{SPXY} = \begin{bmatrix} \Delta\text{SPX} \\ \Delta\text{SPY} \end{bmatrix}$	A change in the slant plane projection point in slant plane coordinates SPX and SPY (in meters). See Section 11.2.
$\Delta\mathbf{ARP}_{\text{COA}}^{\text{SCP}} = \begin{bmatrix} \Delta\text{ARP_X} \\ \Delta\text{ARP_Y} \\ \Delta\text{ARP_Z} \end{bmatrix}$	A change in ARP position adjustable parameter offset in ECF coordinates (in meters).
$\Delta\mathbf{VARP}_{\text{COA}} = \begin{bmatrix} \Delta\text{VARP_X} \\ \Delta\text{VARP_Y} \\ \Delta\text{VARP_Z} \end{bmatrix}$	A change in ARP velocity adjustable parameter offset in ECF coordinates (in meters/sec).
ΔR_BIAS	A change in range bias adjustable parameter offset (in meters).

The sensitivity matrices that may be computed for a given image/scene projection pair are summarized in Tables 11-2 through 11-5 below. The matrices that relate changes in scene point or image location to slant plane projection point are listed in Table 11-2. The matrices that relate changes in scene point position and changes in image grid location are listed in Table 11-3. For the matrices in Tables 11-2 and 11-3, the Adjustable Parameter Offsets are held constant and are set equal to zero. The matrices that relate changes in Adjustable Parameter Offsets to changes in scene point position are listed in Table 11-4. For the matrices in Table 11-4, the image grid location is held constant. The matrices that relate changes in Adjustable Parameter Offsets to changes in image grid location are listed in Table 11-5. For the matrices in Table 11-5, the scene point position is held constant.

Table 11-2 Slant Plane Point Sensitivity Matrices

Adjustable Parameter Offsets Held Constant

$$\underbrace{\Delta\mathbf{IL} = \begin{bmatrix} \Delta\text{xrow} \\ \Delta\text{ycol} \end{bmatrix}}_{\text{Change In Image Location}} \quad \underbrace{\Delta\mathbf{SPXY} = \begin{bmatrix} \Delta\text{SPX} \\ \Delta\text{SPY} \end{bmatrix}}_{\text{Change In Slant Plane XY Projection Point}} \quad \underbrace{\Delta\mathbf{GPXY} = \begin{bmatrix} \Delta\text{GPX} \\ \Delta\text{GPY} \end{bmatrix}}_{\text{Change In Ground Plane Point Specified In GP Coordinates}} \quad \underbrace{\Delta\mathbf{PT} = \begin{bmatrix} \Delta\text{PT_X} \\ \Delta\text{PT_Y} \\ \Delta\text{PT_Z} \end{bmatrix}}_{\text{Change in Scene Point In ECF Coordinates}}$$

Matrix	Size	Description
$\mathbf{M}_{\text{PT}}^{\text{SPXY}}$	**2 x 3**	Sensitivity Matrix to compute the change in Slant Plane projection point (Δ**SPXY**) due to a change in scene point position in ECF coordinates (Δ**PT**) $$\Delta\mathbf{SPXY} = \mathbf{M}_{\text{PT}}^{\text{SPXY}} \bullet \Delta\mathbf{PT}$$ Constant: All Adjustable Parameter Offsets are constant & set to zero.
$\mathbf{M}_{\text{GPXY}}^{\text{PT}}$	**3 x 2**	Sensitivity Matrix to compute the change in scene point in ECF coordinates (Δ**PT**) due to a change in Ground Plane point position in GP coordinates (Δ**GPXY**) $$\Delta\mathbf{PT} = \mathbf{M}_{\text{GPXY}}^{\text{PT}} \bullet \Delta\mathbf{GPXY}$$ Constant: All Adjustable Parameter Offsets are constant and set to zero.
$\mathbf{M}_{\text{GPXY}}^{\text{SPXY}}$	**2 x 2**	Sensitivity Matrix to compute the change in Slant Plane projection point (Δ**SPXY**) due to a change in Ground Plane point position (Δ**GPXY**) $$\Delta\mathbf{SPXY} = \mathbf{M}_{\text{GPXY}}^{\text{SPXY}} \bullet \Delta\mathbf{GPXY}$$ Constant: All Adjustable Parameter Offsets are constant and set to zero.
$\mathbf{M}_{\text{SPXY}}^{\text{GPXY}}$	**2 x 2**	Sensitivity Matrix to compute the change in Ground Plane point position (Δ**GPXY**) due to a change in Slant Plane projection point (Δ**SPXY**) $$\Delta\mathbf{GPXY} = \mathbf{M}_{\text{SPXY}}^{\text{GPXY}} \bullet \Delta\mathbf{SPXY}$$ Constant: All Adjustable Parameter Offsets are constant & set to zero.
$\mathbf{M}_{\text{IL}}^{\text{SPXY}}$	**2 x 2**	Sensitivity Matrix to compute the change in Slant Plane projection point (Δ**SPXY**) due to a change in Image Location (Δ**IL**) $$\Delta\mathbf{SPXY} = \mathbf{M}_{\text{IL}}^{\text{SPXY}} \bullet \Delta\mathbf{IL}$$ Constant: All Adjustable Parameter Offsets are constant & set to zero.
$\mathbf{M}_{\text{SPXY}}^{\text{IL}}$	**2 x 2**	Sensitivity Matrix to compute the change in Image Location (Δ**IL**) due to a change in Slant Plane projection point (Δ**SPXY**) $$\Delta\mathbf{IL} = \mathbf{M}_{\text{SPXY}}^{\text{IL}} \bullet \Delta\mathbf{SPXY}$$ Constant: All Adjustable Parameter Offsets are constant & set to zero.

Table 11-3 Image To Scene Sensitivity Matrices

Adjustable Parameter Offsets Held Constant

$$\underbrace{\Delta\mathbf{IL} = \begin{bmatrix} \Delta\text{xrow} \\ \Delta\text{ycol} \end{bmatrix}}_{\text{Change In Image Location}} \qquad \underbrace{\Delta\mathbf{GPXY} = \begin{bmatrix} \Delta\text{GPX} \\ \Delta\text{GPY} \end{bmatrix}}_{\substack{\text{Change In Ground Plane XY} \\ \text{Projection Point}}} \qquad \underbrace{\Delta\mathbf{PT} = \begin{bmatrix} \Delta\text{PT_X} \\ \Delta\text{PT_Y} \\ \Delta\text{PT_Z} \end{bmatrix}}_{\substack{\text{Change in Scene Point} \\ \text{In ECF Coordinates}}} \qquad \underbrace{\Delta\text{HAE}}_{\substack{\text{Change In Scene HAE} \\ \text{For } \mathbf{uGPN} = \mathbf{uUP0}}}$$

Matrix	Size	Description
$\mathbf{M}_{\text{PT}}^{\text{IL}}$	**2 x 3**	Sensitivity Matrix to compute the change in Image Location (Δ**IL**) due to a change in scene point position in ECF coordinates (Δ**PT**) $\Delta\mathbf{IL} = \mathbf{M}_{\text{PT}}^{\text{IL}} \bullet \Delta\mathbf{PT}$ Constant: All Adjustable Parameter Offsets are constant & set to zero.
$\mathbf{M}_{\text{GPXY}}^{\text{IL}}$	**2 x 2**	Sensitivity Matrix to compute the change in Image Location (Δ**IL**) due to a change in scene point position where the scene point is constrained to be in the ground plane (Δ**GPXY**) $\Delta\mathbf{IL} = \mathbf{M}_{\text{GPXY}}^{\text{IL}} \bullet \Delta\mathbf{GPXY}$ Constant: All Adjustable Parameter Offsets are constant & set to zero.
$\mathbf{M}_{\text{IL}}^{\text{GPXY}}$	**2 x 2**	Sensitivity Matrix to compute the change in Ground Plane point position (Δ**GPXY**) due to a change in Image Location (Δ**IL**) $\Delta\mathbf{GPXY} = \mathbf{M}_{\text{IL}}^{\text{GPXY}} \bullet \Delta\mathbf{IL}$ Constant: All Adjustable Parameter Offsets constant & set to zero.
$\mathbf{M}_{\text{HAE}}^{\text{PT}}$	**3 x 1**	Sensitivity Matrix to compute the change in scene point position in ECF coordinates (Δ**PT**) due to a change in scene point HAE (ΔHAE) $\Delta\mathbf{PT} = \mathbf{M}_{\text{HAE}}^{\text{PT}} \bullet \Delta\text{HAE}$ Constant: Image grid location **IL0**. All Adjustable Parameter Offsets constant & set to zero.
$\mathbf{M}_{\text{HAE}}^{\text{IL}}$	**2 x 1**	Sensitivity Matrix to compute the change in Image Location (Δ**IL**) due to a change in scene point HAE (ΔHAE) $\Delta\mathbf{IL} = \mathbf{M}_{\text{HAE}}^{\text{IL}} \bullet \Delta\text{HAE}$ Constant: Scene point Lat & Lon are held constant. All Adjustable Parameter Offsets constant & set to zero.

Table 11-4 Adjustable Parameters / Scene Point Sensitivity Matrices

Image Grid Location Held Constant

$$\underbrace{\Delta\mathbf{ARP}_{\mathrm{COA}}^{\mathrm{SCP}} = \begin{bmatrix} \Delta\mathrm{ARP_SCP_X} \\ \Delta\mathrm{ARP_SCP_Y} \\ \Delta\mathrm{ARP_SCP_Z} \end{bmatrix}}_{\substack{\text{Change in ARP Position Offset} \\ \text{ECF Coordinates}}} \qquad \underbrace{\Delta\mathbf{VARP}_{\mathrm{COA}} = \begin{bmatrix} \Delta\mathrm{VARP_X} \\ \Delta\mathrm{VARP_Y} \\ \Delta\mathrm{VARP_Z} \end{bmatrix}}_{\substack{\text{Change in ARP Velocity Offset} \\ \text{ECF Coordinates}}} \qquad \underbrace{\Delta\mathrm{R_BIAS}}_{\substack{\text{Change In COA} \\ \text{Range Bias Offset}}}$$

Matrix	Size	Description
$\mathbf{MIL}_{\Delta\mathrm{ARP}}^{\mathrm{SPXY}}$	**2 x 3**	Sensitivity Matrix to compute the change in Slant Plane projection point (Δ**SPXY**) due to a change in ARP position adjustable parameter offset in ECF coordinates ($\Delta\mathbf{ARP}_{\mathrm{COA}}^{\mathrm{SCP}}$) $$\Delta\mathbf{SPXY} = \mathbf{MIL}_{\Delta\mathrm{ARP}}^{\mathrm{SPXY}} \bullet \Delta\mathbf{ARP}_{\mathrm{COA}}^{\mathrm{SCP}}$$ Constant: Image grid location **IL0**. All other adjustable parameter offsets are constant and set to zero.
$\mathbf{MIL}_{\Delta\mathrm{VARP}}^{\mathrm{SPXY}}$	**2 x 3**	Sensitivity Matrix to compute the change in Slant Plane projection point (Δ**SPXY**) due to a change in ARP velocity adjustable parameter offset in ECF coordinates ($\Delta\mathbf{VARP}_{\mathrm{COA}}$) $$\Delta\mathbf{SPXY} = \mathbf{MIL}_{\Delta\mathrm{VARP}}^{\mathrm{SPXY}} \bullet \Delta\mathbf{VARP}_{\mathrm{COA}}$$ Constant: Image grid location **IL0**. All other adjustable parameter offsets are constant and set to zero.
$\mathbf{MIL}_{\Delta\mathrm{R}}^{\mathrm{SPXY}} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$	**2 x 1**	Sensitivity Matrix to compute the change in Slant Plane projection point (Δ**SPXY**) due to a change in COA range adjustable parameter offset (ΔR_BIAS) $$\Delta\mathbf{SPXY} = \mathbf{MIL}_{\Delta\mathrm{R}}^{\mathrm{SPXY}} \bullet \Delta\mathrm{R_BIAS}$$ Constant: Image grid location **IL0**. All other adjustable parameter offsets are constant and set to zero.
$\mathbf{MIL}_{\Delta\mathrm{ARP}}^{\mathrm{GPXY}}$	**2 x 3**	Sensitivity Matrix to compute the change in scene point ground plane position (Δ**GPXY**) due to a change in ARP position adjustable parameter offset in ECF coordinates ($\Delta\mathbf{ARP}_{\mathrm{COA}}^{\mathrm{SCP}}$) $$\Delta\mathbf{GPXY} = \mathbf{MIL}_{\Delta\mathrm{ARP}}^{\mathrm{GPXY}} \bullet \Delta\mathbf{ARP}_{\mathrm{COA}}^{\mathrm{SCP}}$$ Constant: Image grid location **IL0**. All other adjustable parameter offsets are constant and set to zero.
$\mathbf{MIL}_{\Delta\mathrm{VARP}}^{\mathrm{GPXY}}$	**2 x 3**	Sensitivity Matrix to compute the change in scene point ground plane position (Δ**GPXY**) due to a change in ARP velocity adjustable parameter offset in ECF coordinates ($\Delta\mathbf{VARP}_{\mathrm{COA}}$) $$\Delta\mathbf{GPXY} = \mathbf{MIL}_{\Delta\mathrm{VARP}}^{\mathrm{GPXY}} \bullet \Delta\mathbf{VARP}_{\mathrm{COA}}$$ Constant: Image grid location **IL0**. All other adjustable parameter offsets are constant and set to zero.

$\mathbf{MIL}_{\Delta R}^{GPXY}$	**2 x 1**	Sensitivity Matrix to compute the change in scene point ground plane position (Δ**GPXY**) due to a change in COA range adjustable parameter offset (ΔR_BIAS) $$\Delta\mathbf{GPXY} = \mathbf{MIL}_{\Delta R}^{GPXY} \bullet \Delta R_BIAS$$ Constant: Image grid location **IL0**. All other adjustable parameter offsets are constant and set to zero.

Table 11-5 Adjustable Parameters / Image Location Sensitivity Matrices		
Scene Point Position Held Constant		
$\underbrace{\Delta\mathbf{ARP}_{COA}^{SCP} = \begin{bmatrix} \Delta ARP_SCP_X \\ \Delta ARP_SCP_Y \\ \Delta ARP_SCP_Z \end{bmatrix}}_{\substack{\text{Change in ARP Position Offset} \\ \text{ECF Coordinates}}}$ $\underbrace{\Delta\mathbf{VARP}_{COA} = \begin{bmatrix} \Delta VARP_X \\ \Delta VARP_Y \\ \Delta VARP_Z \end{bmatrix}}_{\substack{\text{Change in ARP Velocity Offset} \\ \text{ECF Coordinates}}}$ $\underbrace{\Delta R_BIAS}_{\substack{\text{Change In COA} \\ \text{Range Bias Offset}}}$		
Matrix	**Size**	**Description**
$\mathbf{MPT}_{\Delta ARP}^{SPXY}$	**2 x 3**	Sensitivity Matrix to compute the change in Slant Plane projection point (Δ**SPXY**) due to a change in ARP position adjustable parameter offset in ECF coordinates ($\Delta\mathbf{ARP}_{COA}^{SCP}$) $$\Delta\mathbf{SPXY} = \mathbf{MPT}_{\Delta ARP}^{SPXY} \bullet \Delta\mathbf{ARP}_{COA}^{SCP}$$ Constant: Scene point position **PT0**. All other adjustable parameter offsets are constant and set to zero.
$\mathbf{MPT}_{\Delta VARP}^{SPXY}$	**2 x 3**	Sensitivity Matrix to compute the change in Slant Plane projection point (Δ**SPXY**) due to a change in ARP velocity adjustable parameter offset in ECF coordinates ($\Delta\mathbf{VARP}_{COA}$) $$\Delta\mathbf{SPXY} = \mathbf{MPT}_{\Delta VARP}^{SPXY} \bullet \Delta\mathbf{VARP}_{COA}$$ Constant: Scene point position **PT0**. All other adjustable parameter offsets are constant and set to zero.
$\mathbf{MPT}_{\Delta R}^{SPXY} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$	**2 x 1**	Sensitivity Matrix to compute the change in Slant Plane projection point (Δ**SPXY**) due to a change in COA range adjustable parameter offset (ΔR_BIAS) $$\Delta\mathbf{SPXY} = \mathbf{MPT}_{\Delta R}^{SPXY} \bullet \Delta R_BIAS$$ Constant: Scene point position **PT0**. All other adjustable parameter offsets are constant and set to zero.
$\mathbf{MPT}_{\Delta ARP}^{IL}$	**2 x 3**	Sensitivity Matrix to compute the change in Image Location (Δ**IL**) due to a change in ARP position adjustable parameter offset in ECF coordinates ($\Delta\mathbf{ARP}_{COA}^{SCP}$) $$\Delta\mathbf{IL} = \mathbf{MPT}_{\Delta ARP}^{IL} \bullet \Delta\mathbf{ARP}_{COA}^{SCP}$$ Constant: Scene point position **PT0**. All other adjustable parameter offsets are constant and set to zero.

$\mathbf{MPT}^{\mathrm{IL}}_{\Delta\mathrm{VARP}}$	**2 x 3**	Sensitivity Matrix to compute the change in Image Location (Δ**IL**) due to a change in ARP velocity adjustable parameter offset in ECF coordinates ($\Delta\mathbf{VARP}_{\mathrm{COA}}$) $$\Delta\mathbf{IL} = \mathbf{MPT}^{\mathrm{IL}}_{\Delta\mathrm{VARP}} \bullet \Delta\mathbf{VARP}_{\mathrm{COA}}$$ Constant: Scene point position **PT0**. All other adjustable parameter offsets are constant and set to zero.
$\mathbf{MPT}^{\mathrm{IL}}_{\Delta\mathrm{R}}$	**2 x 1**	Sensitivity Matrix to compute the change in Image Location (Δ**IL**) due to a change in COA range adjustable parameter offset (ΔR_BIAS) $$\Delta\mathbf{IL} = \mathbf{MPT}^{\mathrm{IL}}_{\Delta\mathrm{R}} \bullet \Delta\mathrm{R_BIAS}$$ Constant: Scene point position **PT0**. All other adjustable parameter offsets are constant and set to zero.

For an image grid location **IL** expressed in image coordinates (xrow, ycol), let image grid location **IIDX** be the same grid location expressed in image indices (irow, icol). Grid locations **IL** and **IIDX** are related by matrices the $\mathbf{M}^{\mathrm{IL}}_{\mathrm{IIDX}}$ and $\mathbf{M}^{\mathrm{IIDX}}_{\mathrm{IL}}$. The image row spacing, Row_SS, and image column spacing, Col_SS, are in meters.

$$\mathbf{IL} = \begin{bmatrix} \mathrm{xrow} \\ \mathrm{ycol} \end{bmatrix} \Leftrightarrow \mathbf{IIDX} = \begin{bmatrix} \mathrm{irow} \\ \mathrm{icol} \end{bmatrix} \qquad \mathbf{IL} = \mathbf{M}^{\mathrm{IL}}_{\mathrm{IIDX}} \bullet \mathbf{IIDX} \;\; \& \;\; \mathbf{IIDX} = \mathbf{M}^{\mathrm{IIDX}}_{\mathrm{IL}} \bullet \mathbf{IL}$$

$$\mathbf{M}^{\mathrm{IL}}_{\mathrm{IIDX}} = \begin{bmatrix} \mathrm{Row_SS} & 0 \\ 0 & \mathrm{Col_SS} \end{bmatrix} \qquad \mathbf{M}^{\mathrm{IIDX}}_{\mathrm{IL}} = \begin{bmatrix} 1/\mathrm{Row_SS} & 0 \\ 0 & 1/\mathrm{Col_SS} \end{bmatrix}$$

For applications that work with image locations in pixel coordinates, the sensitivity matrices may be scaled to yield changes in image location in pixel indices, $\Delta\mathbf{IIDX} = \mathbf{M}^{\mathrm{IIDX}}_{\mathrm{IL}} \bullet \Delta\mathbf{IL}$. For a sensitivity matrix for which image grid location **IL** is the dependent parameter, the matrix is pre-multiplied by $\mathbf{M}^{\mathrm{IIDX}}_{\mathrm{IL}}$ to yield the image location sensitivity in pixels. For a sensitivity matrix for which image grid location is the independent parameter, the matrix is post-multiplied by $\mathbf{M}^{\mathrm{IL}}_{\mathrm{IIDX}}$ to allow image changes in pixel coordinates. For example, the matrices relating changes in image grid location and ground plane position are modified as follows.

Δ**IIDX** Dependent: $$\mathbf{M}^{\mathrm{IIDX}}_{\mathrm{GPXY}} = \underbrace{\mathbf{M}^{\mathrm{IIDX}}_{\mathrm{IL}} \bullet \mathbf{M}^{\mathrm{IL}}_{\mathrm{GPXY}}}_{\text{Scale output from meters to pixels.}} \;\Rightarrow\; \Delta\mathbf{IIDX} = \mathbf{M}^{\mathrm{IIDX}}_{\mathrm{GPXY}} \bullet \Delta\mathbf{GPXY}$$

Δ**IIDX** Independent: $$\mathbf{M}^{\mathrm{GPXY}}_{\mathrm{IIDX}} = \underbrace{\mathbf{M}^{\mathrm{GPXY}}_{\mathrm{IL}} \bullet \mathbf{M}^{\mathrm{IL}}_{\mathrm{IIDX}}}_{\text{Scale input from pixels to meters.}} \;\Rightarrow\; \Delta\mathbf{GPXY} = \mathbf{M}^{\mathrm{GPXY}}_{\mathrm{IIDX}} \bullet \Delta\mathbf{IIDX}$$

In the sections that follow, scene point positions **PT** are specified in ECF coordinates except where noted. All are expressed in units of meters. Image grid locations **IL** are specified in image coordinates (xrow, ycol). Image coordinates are also in units of meters. Refer to Section 2.3 for the conventions used for all SICD products.

11.1 Image/Scene Pair Projection Parameters

For a given image/scene projection pair, the projection sensitivity matrices are computed for the projection geometry with all adjustable parameter offsets set to zero. An example projection pair **IL0** and **PT0** are shown in Figure 11-2. For image grid location **IL0**, the COA projection set is computed from the image metadata. The COA projection set defines the R/Rdot contour that intersects the scene surface at scene point **PT0**. The COA projection set is a set of 6 parameters computed for location **IL0**. See Section 4.

$$\text{Image Location: } \mathbf{IL0} = \begin{bmatrix} \text{xrow0} \\ \text{ycol0} \end{bmatrix} \qquad \text{Scene Point (ECF): } \mathbf{PT0} = \begin{bmatrix} \text{PT0_X} \\ \text{PT0_Y} \\ \text{PT0_Z} \end{bmatrix}$$

$$\text{Projection Set: } \left\{ t0_{COA} \quad t_{COA}^{SCP} \quad \mathbf{ARP0}_{COA} \quad \mathbf{VARP0}_{COA} \quad R0_{COA} \quad Rdot0_{COA} \right\}$$

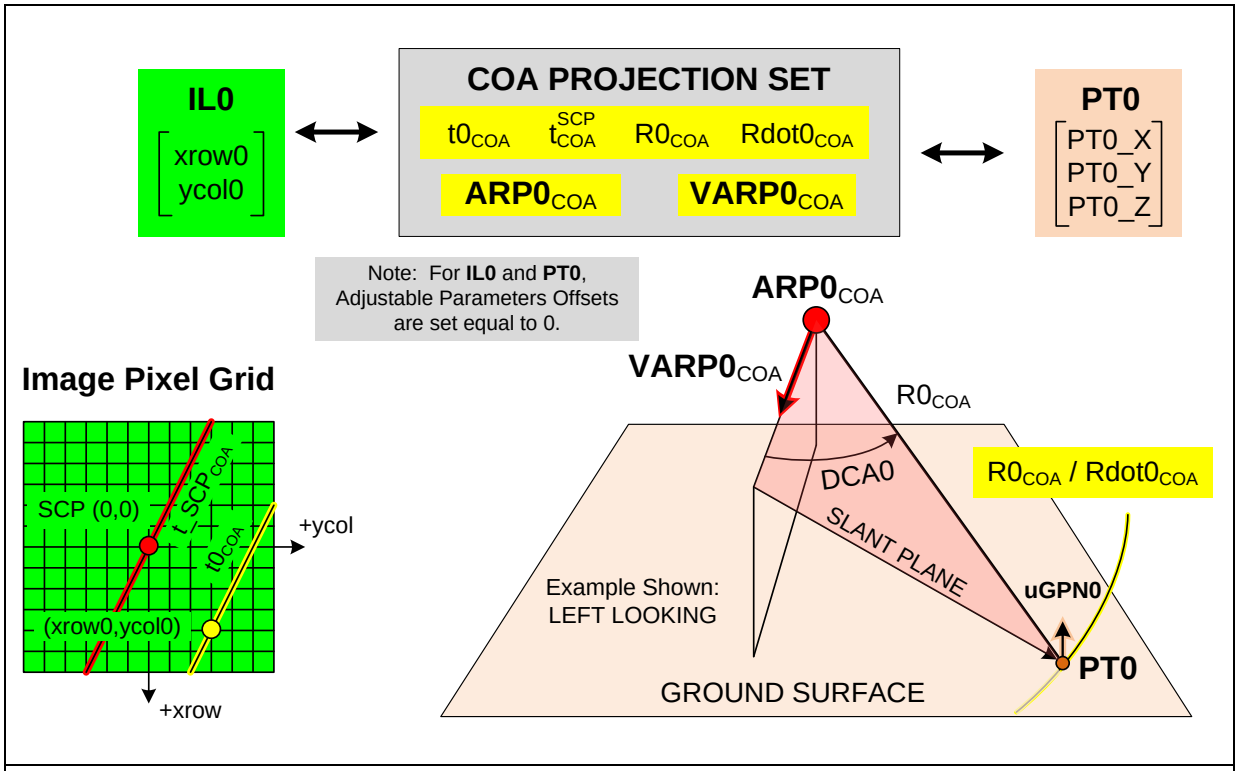


Figure 11-2 Image/Scene Projection Pair IL0 & PT0
Projection geometry used for computing projection sensitivity parameters for IL0.

The point **PT0** position can also be expressed in WGS 84 geodetic coordinates. Point **PT0** is at height PT0_HAE above the WGS 84 ellipsoid. The unit vector **uUP0** is the normal to the surface of constant HAE at point **PT0**. The unit vector **uGPN0** is the normal to the surface model at point **PT0**. The surface normal is always oriented upward and away from the center of the earth.

$$\underbrace{\begin{bmatrix} PT0_X \\ PT0_Y \\ PT0_Z \end{bmatrix}}_{\substack{\text{ECF} \\ \text{Coordinates}}} \Rightarrow \underbrace{\begin{bmatrix} PT0_Lat \\ PT0_Lon \\ PT0_HAE \end{bmatrix}}_{\text{WGS 84 LLH}} \qquad \underbrace{\mathbf{uUP0} = \begin{bmatrix} \cos(PT0_Lon)\bullet\cos(PT0_Lat) \\ \sin(PT0_Lon)\bullet\cos(PT0_Lat) \\ \sin(PT0_Lat) \end{bmatrix}}_{\substack{\text{Unit vector normal to the} \\ \text{surface of constant HAE at PT0}}}$$

$$\mathbf{uGPN0} = \begin{bmatrix} uGPN_X \\ uGPN_Y \\ uGPN_Z \end{bmatrix} \qquad \mathbf{uGPN0} \text{ Oriented "Upward": } \underbrace{\mathbf{uGPN0} \bullet \mathbf{uUP0}}_{\text{Vector Dot Product}} > 0$$

11.2 Ground Plane & Slant Plane Coordinates

For the image/scene projection pair **IL0** and **PT0**, define a set of Ground Plane Coordinates (GPC) and a set of Slant Plane Coordinates (SPC) both with origin at **PT0**. See Figure 11-3. The ground plane coordinates are (GPX, GPY, GPZ). The ground plane is tangent to the surface model at point **PT0**. The slant plane coordinates are (SPX, SPY, SPZ). The COA slant plane at **PT0** contains the COA ARP position and velocity. The GPC and SPC systems are right-handed Cartesian frames.

(1) Compute the unit basis vectors for the Ground Plane Coordinates, **uGPX**, **uGPY** and **uGPZ**. All vectors are in ECF coordinates. Also compute the sine and cosine of the grazing angle, sin_GRAZ and cos_GRAZ. For reference, see SICD Design & Implementation Description Document, Volume 1, Section 4.9. Note: For all geometries, the grazing angle is on the interval from 0 to 90 degrees.

$$\mathbf{uGPZ} = \mathbf{uGPN0} \qquad \mathbf{uGPZ} \text{ is normal to the surface at point PT0}$$

$$ARP0_GPZ = \left(\mathbf{ARP0}_{COA} - \mathbf{PT0}\right) \bullet \mathbf{uGPZ} \qquad sin_GRAZ = \frac{ARP0_GPZ}{R0_{COA}}$$

$$\mathbf{AGPN0} = \mathbf{ARP0}_{COA} - ARP0_GPZ \bullet \mathbf{uGPZ}$$

$$ARP0_GPX = \left|\mathbf{AGPN0} - \mathbf{PT0}\right| \qquad cos_GRAZ = \frac{ARP0_GPX}{R0_{COA}}$$

$$\mathbf{uGPX} = \frac{1}{ARP0_GPX} \bullet \left(\mathbf{AGPN0} - \mathbf{PT0}\right) \qquad \mathbf{uGPY} = \mathbf{uGPZ} \times \mathbf{uGPX}$$

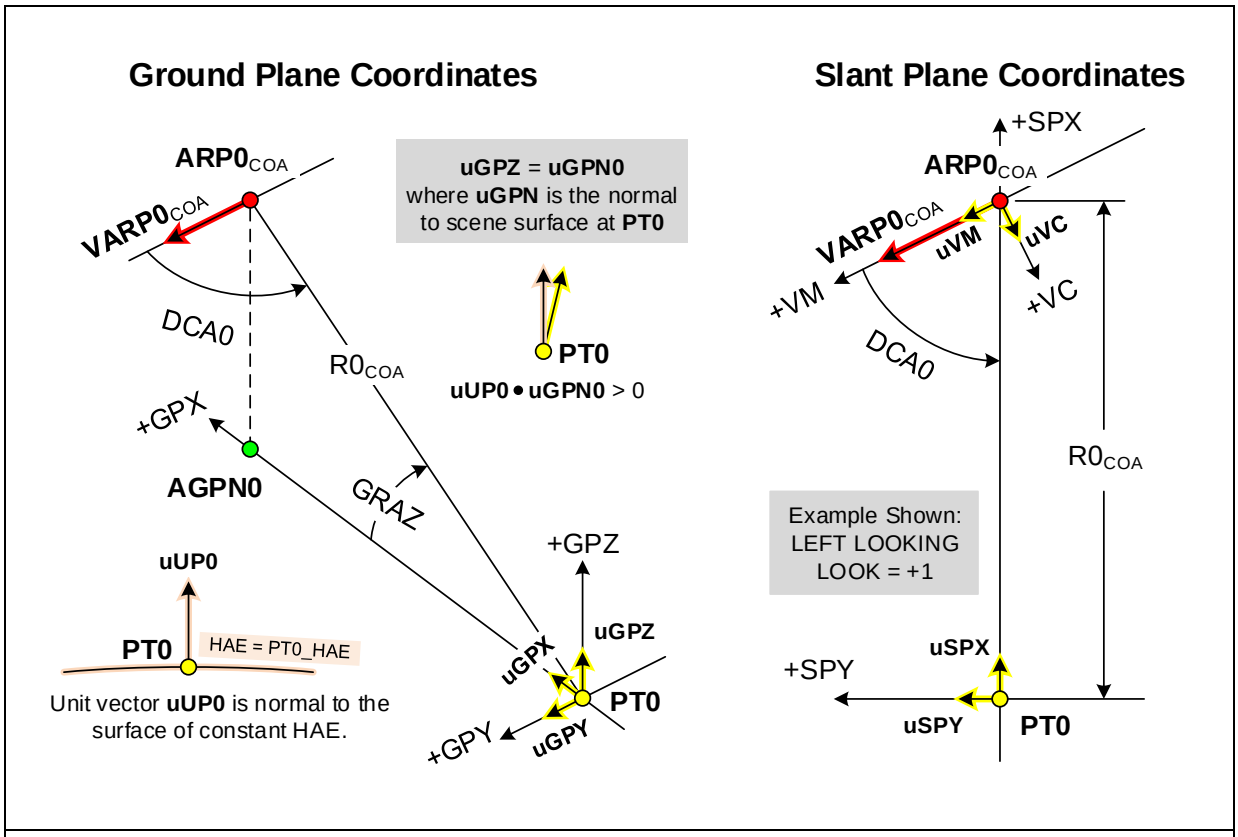


Figure 11-3 Scene Point Centered Coordinates
Ground Plane Coordinates (GPC) and Slant Plane Coordinates (SPC) centered at point PT0.

(2) Compute the unit basis vectors for the Slant Plane Coordinates, **uSPX**, **uSPY** and **uSPZ**. The slant plane normal, **uSPZ**, is always oriented "up" ($\mathbf{uSPZ} \bullet \mathbf{uUP0} > 0$). Unit vectors **uVM** and **uVC** are also in the slant plane.

$$\mathrm{VM0} = \left|\mathbf{VARP0}_{\mathrm{COA}}\right| \qquad \mathbf{uVM} = \frac{1}{\mathrm{VM0}} \bullet \mathbf{VARP0}_{\mathrm{COA}}$$

$$\mathbf{uSPX} = \frac{1}{\mathrm{R0}_{\mathrm{COA}}} \bullet \left(\mathbf{ARP0}_{\mathrm{COA}} - \mathbf{PT0}\right) \qquad \mathbf{SPZ} = \mathrm{LOOK} \bullet \left(\mathbf{uSPX} \times \mathbf{uVM}\right)$$

$$\mathbf{uSPZ} = \frac{1}{\left|\mathbf{SPZ}\right|} \bullet \mathbf{SPZ} \qquad \mathbf{uSPY} = \mathbf{uSPZ} \times \mathbf{uSPX} \qquad \mathbf{uVC} = \mathbf{uSPZ} \times \mathbf{uVM}$$

(3) Compute the sine and cosine of the twist angle, sin_TWST and cos_TWST. Also compute the sine and cosine of the Doppler Cone Angle (DCA), sin_DCA0 and cos_DCA0. For all geometries, the twist angle is on the interval from -90 to 90 degrees and the Doppler Cone Angle is on the interval from 0 to 180 degrees.

$$\mathrm{sin_TWST} = -\mathbf{uGPY} \bullet \mathbf{uSPZ} \qquad \mathrm{cos_TWST} = \mathbf{uGPY} \bullet \mathbf{uSPY}$$

$$\text{cos_DCA0} = -\frac{\text{Rdot0}_{\text{COA}}}{\text{VM0}} \qquad \text{sin_DCA0} = +\left(1 - \text{cos_DCA0}^2\right)^{1/2}$$

11.3 Slant Plane Sensitivity Matrices

The sensitivity matrices that relate changes in image location or scene point position to slant plane projection point are listed in Table 11.2 above. The method for computing them is described below.

Scene Point ⇔ Slant Plane Projection Point: $\mathbf{M}_{\text{PT}}^{\text{SPXY}}$ $\mathbf{M}_{\text{GPXY}}^{\text{PT}}$ $\mathbf{M}_{\text{GPXY}}^{\text{SPXY}}$ $\mathbf{M}_{\text{SPXY}}^{\text{GPXY}}$

(1) Form the 2 x 3 sensitivity matrix to compute the change in slant plane projection point (Δ**SPXY**) due to a change in scene point position in ECF coordinates (Δ**PT**). Form the matrix from the slant plane unit vectors **uSPX** and **uSPY**. Also form the 3 x 2 matrix that converts a change in ground plane point (Δ**GPXY**) expressed in ground plane coordinates GPX and GPY to be expressed in ECF coordinates(Δ**PT**).

$$\mathbf{M}_{\text{PT}}^{\text{SPXY}} = \begin{bmatrix} \mathbf{uSPX}^{\text{T}} \\ \mathbf{uSPY}^{\text{T}} \end{bmatrix} = \begin{bmatrix} \text{uSPX_X} & \text{uSPX_Y} & \text{uSPX_Z} \\ \text{uSPY_X} & \text{uSPY_Y} & \text{uSPY_Z} \end{bmatrix}$$

$$\mathbf{M}_{\text{GPXY}}^{\text{PT}} = \begin{bmatrix} \mathbf{uGPX} & \mathbf{uGPY} \end{bmatrix} = \begin{bmatrix} \text{uGPX_X} & \text{uGPY_X} \\ \text{uGPX_Y} & \text{uGPY_Y} \\ \text{uGPX_Z} & \text{uGPY_Z} \end{bmatrix}$$

(2) Compute the 2 x 2 sensitivity matrix to compute the change in slant plane projection point (Δ**SPXY**) due to a change in scene point position ground plane, Δ**GPXY**. The scene point is constrained to lie in the ground plane.

$$\mathbf{M}_{\text{GPXY}}^{\text{SPXY}} = \begin{bmatrix} \text{cos_GRAZ} & 0 \\ -\text{sin_GRAZ} \bullet \text{sin_TWST} & \text{cos_TWST} \end{bmatrix}$$

(3) Compute the 2 x 2 sensitivity matrix to compute the change in ground plane point (Δ**GPXY**) due to a change slant plane projection point, Δ**SPXY**. The scene point is constrained to lie in the ground plane.

$$\mathbf{M}_{\text{SPXY}}^{\text{GPXY}} = \left\{\mathbf{M}_{\text{GPXY}}^{\text{SPXY}}\right\}^{-1} = \begin{bmatrix} \dfrac{1}{\text{cos_GRAZ}} & 0 \\ \text{tan_GRAZ} \bullet \text{tan_TWST} & \dfrac{1}{\text{cos_TWST}} \end{bmatrix}$$

Image Location ⇔ Slant Plane Projection Point: $\mathbf{M}_{\mathrm{IL}}^{\mathrm{SPXY}}$ $\mathbf{M}_{\mathrm{SPXY}}^{\mathrm{IL}}$

The sensitivity matrix that relates a change in image location Δ**IL** to a change in slant plane projection point Δ**SPXY** is computed as follows. See Figure 11-4 and Figure 11-5. First, the change in slant plane point, Δ**SPXY1x**, due to a change in the row coordinate, Δxrow, is computed with the column coordinate held constant. Second, the change in slant plane point, Δ**SPXY1y**, due to a change in the column coordinate, Δycol, is computed with the row coordinate held constant. The sensitivity matrix $\mathbf{M}_{\mathrm{IL}}^{\mathrm{SPXY}}$ is a 2 x 2 matrix formed from the computed changes in slant plane points. The sensitivity matrix $\mathbf{M}_{\mathrm{SPXY}}^{\mathrm{IL}}$ is computed as the inverse of the matrix $\mathbf{M}_{\mathrm{IL}}^{\mathrm{SPXY}}$.

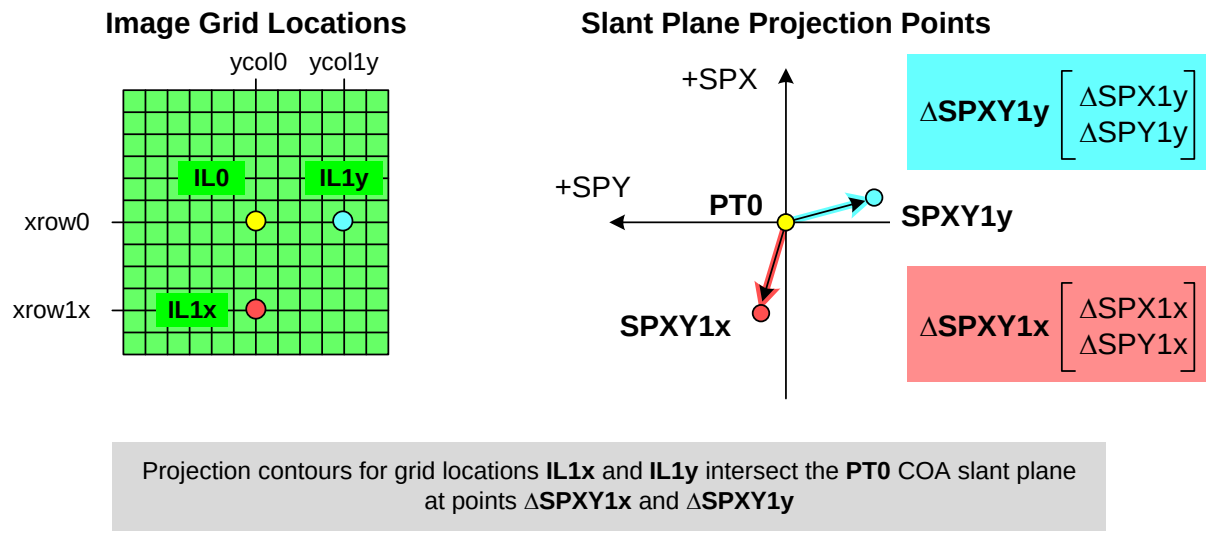


Figure 11-4 Image Locations IL1x & IL1y
Image locations IL1x and IL1y used to compute the slant plane projection points.

ΔSPXY Due To a Change in the Row Coordinate

(1) Select an increment in the row coordinate, Δxrow. Compute image grid location **IL1x** from location **IL0** as shown. Compute the COA projection set for image grid location **IL1x** using only the image metadata. Suggested value: $\Delta\mathrm{xrow} = \mathrm{Min}\left(\mathrm{Row_SS},\ 1.0\ \mathrm{m}\right)$.

$$\mathbf{IL1x} = \mathbf{IL0} + \begin{bmatrix} \Delta\mathrm{xrow} \\ 0 \end{bmatrix} = \begin{bmatrix} \mathrm{xrow0} + \Delta\mathrm{xrow} \\ \mathrm{ycol0} \end{bmatrix}$$

COA Projection Set: $\mathbf{IL1x} \rightarrow \begin{Bmatrix} \mathrm{t1x}_{\mathrm{COA}} & \mathbf{ARP1x}_{\mathrm{COA}} & \mathrm{R1x}_{\mathrm{COA}} \\ \mathrm{t}_{\mathrm{COA}}^{\mathrm{SCP}} & \mathbf{VARP1x}_{\mathrm{COA}} & \mathrm{Rdot1x}_{\mathrm{COA}} \end{Bmatrix}$

(2) Compute the change in COA projection contour parameters for image location **IL1x** relative to **IL0**.

$$\Delta\mathbf{ARP1x}_{\mathrm{COA}} = \mathbf{ARP1x}_{\mathrm{COA}} - \mathbf{ARP0}_{\mathrm{COA}} \qquad \Delta\mathrm{R1x}_{\mathrm{COA}} = \mathrm{R1x}_{\mathrm{COA}} - \mathrm{R0}_{\mathrm{COA}}$$

$$\Delta\mathbf{VARP1x}_{\mathrm{COA}} = \mathbf{VARP1x}_{\mathrm{COA}} - \mathbf{VARP0}_{\mathrm{COA}} \qquad \Delta\mathrm{Rdot1x}_{\mathrm{COA}} = \mathrm{Rdot1x}_{\mathrm{COA}} - \mathrm{Rdot0}_{\mathrm{COA}}$$

(3) Compute the change in COA position in the SPX and SPY directions. Also, compute the change in COA velocity in the VM and VC directions.

$$\Delta\mathrm{ARP1x_SPX} = \Delta\mathbf{ARP1x}_{\mathrm{COA}} \bullet \mathbf{uSPX} \qquad \Delta\mathrm{VM1x} = \Delta\mathbf{VARP1x}_{\mathrm{COA}} \bullet \mathbf{uVM}$$

$$\Delta\mathrm{ARP1x_SPY} = \Delta\mathbf{ARP1x}_{\mathrm{COA}} \bullet \mathbf{uSPY} \qquad \Delta\mathrm{VC1x} = \Delta\mathbf{VARP1x}_{\mathrm{COA}} \bullet \mathbf{uVC}$$

(4) Compute the change in the cosine of the DCA, Δcos_DCA1x, due to the change in the magnitude of the velocity and the COA range rate, ΔVM1x and $\Delta\mathrm{Rdot1x}_{\mathrm{COA}}$. Also, compute the change in DCA, ΔDCA1x. Parameter ΔDCA1x is in radians. A positive ΔDCA1x is a counter clockwise rotation for left looking collections and a clockwise rotation for right looking collections.

$$\Delta\mathrm{cos_DCA1x} = -\frac{1}{\mathrm{VM0}}\left(\Delta\mathrm{Rdot1x}_{\mathrm{COA}} + \Delta\mathrm{VM1x} \bullet \mathrm{cos_DCA0}\right)$$

$$\Delta\mathrm{DCA1x} = -\frac{1}{\mathrm{sin_DCA0}} \bullet \Delta\mathrm{cos_DCA1x}$$

(5) Compute the rotation of the velocity vector in the slant plane, ΔVDIR1x, due to the cross velocity component ΔVC1x. Parameter ΔVDIR1x is in radians.

$$\Delta\mathrm{VDIR1x} = \frac{1}{\mathrm{VM0}} \bullet \Delta\mathrm{VC1x} \qquad \Delta\mathrm{VDIR1x} > 0 \Leftrightarrow \text{CCW Rotation}$$

(6) Compute the rotation in the slant plane projection direction, ΔANG1x due to the change in velocity direction and the change in the DCA.

$$\Delta\mathrm{ANG1x} = \Delta\mathrm{VDIR1x} + \mathrm{LOOK} \bullet \Delta\mathrm{DCA1x} \qquad \Delta\mathrm{ANG1x} > 0 \Leftrightarrow \text{CCW Rotation}$$

(7) Compute the change in the slant plane projection point, Δ**SPXY1x**, due to the change in image row coordinate, Δxrow. Parameters ΔSPX1x and ΔSPY1x are in meters. Note: Small angle approximations are used for cosine and sine of ΔANG1x.

$$\Delta\mathrm{SPX1x} = \Delta\mathrm{ARP1x_SPX} - \Delta\mathrm{R1x}_{\mathrm{COA}}$$

$$\Delta\mathrm{SPY1x} = \Delta\mathrm{ARP1x_SPY} - \mathrm{R0}_{\mathrm{COA}} \bullet \Delta\mathrm{ANG1x}$$

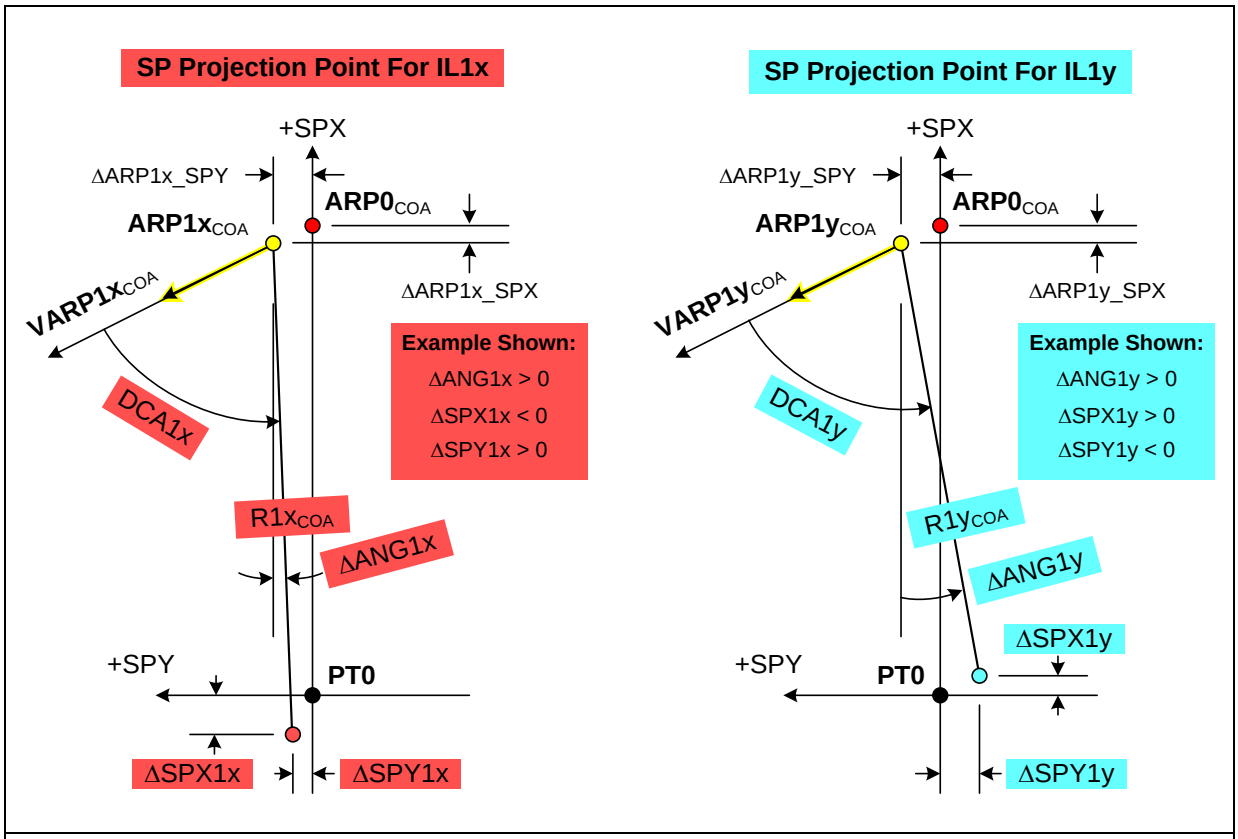


Figure 11-5 Slant Plane Projection Points

Slant plane projection points for IL1x and IL1y .

ΔSPXY Due To a Change in the Column Coordinate

(1) Select an increment in the column coordinate, Δycol. Compute image grid location **IL1y** from location **IL0** as shown. Compute the COA projection set for image grid location **IL1y** using only the image metadata. Suggested value: $\Delta ycol = \mathrm{Min}\left(\mathrm{Col_SS},\ 1.0\ \mathrm{m}\right)$.

$$\mathbf{IL1y} = \mathbf{IL0} + \begin{bmatrix} 0 \\ \Delta ycol \end{bmatrix} = \begin{bmatrix} xrow0 \\ ycol0 + \Delta ycol \end{bmatrix}$$

$$\text{COA Projection Set: } \mathbf{IL1y} \Rightarrow \begin{Bmatrix} t1y_{COA} & \mathbf{ARP1y}_{COA} & R1y_{COA} \\ t_{COA}^{SCP} & \mathbf{VARP1y}_{COA} & Rdot1y_{COA} \end{Bmatrix}$$

In the following description, Steps 2 through 7 are used to compute parameters ΔSPX1y and ΔSPY1y. These are the same computations described in Steps 2 through 7 above to compute parameters ΔSPX1x and ΔSPY1x.

(2) Compute the change in COA projection contour parameters for image location **IL1y** relative to **IL0**.

$$\Delta\mathbf{ARP1y}_{\mathrm{COA}} = \mathbf{ARP1y}_{\mathrm{COA}} - \mathbf{ARP0}_{\mathrm{COA}} \qquad \Delta\mathrm{R1y}_{\mathrm{COA}} = \mathrm{R1y}_{\mathrm{COA}} - \mathrm{R0}_{\mathrm{COA}}$$

$$\Delta\mathbf{VARP1y}_{\mathrm{COA}} = \mathbf{VARP1y}_{\mathrm{COA}} - \mathbf{VARP0}_{\mathrm{COA}} \qquad \Delta\mathrm{Rdot1y}_{\mathrm{COA}} = \mathrm{Rdot1y}_{\mathrm{COA}} - \mathrm{Rdot0}_{\mathrm{COA}}$$

(3) Compute the change in COA position in the SPX and SPY directions. Also, compute the change in COA velocity in the VM and VC directions.

$$\Delta\mathrm{ARP1y_SPX} = \Delta\mathbf{ARP1y}_{\mathrm{COA}} \bullet \mathbf{uSPX} \qquad \Delta\mathrm{VM1y} = \Delta\mathbf{VARP1y}_{\mathrm{COA}} \bullet \mathbf{uVM}$$

$$\Delta\mathrm{ARP1y_SPY} = \Delta\mathbf{ARP1y}_{\mathrm{COA}} \bullet \mathbf{uSPY} \qquad \Delta\mathrm{VC1y} = \Delta\mathbf{VARP1y}_{\mathrm{COA}} \bullet \mathbf{uVC}$$

(4) Compute the change in the cosine of the DCA, Δcos_DCA1y, due to the change in magnitude of the velocity and the COA range rate, ΔVM1y and $\Delta\mathrm{Rdot1y}_{\mathrm{COA}}$. Also compute the change in DCA, ΔDCA1y. Parameter ΔDCA1y is in radians.

$$\Delta\mathrm{cos_DCA1y} = -\frac{1}{\mathrm{VM0}}\left(\Delta\mathrm{Rdot1y}_{\mathrm{COA}} + \Delta\mathrm{VM1y} \bullet \mathrm{cos_DCA0}\right)$$

$$\Delta\mathrm{DCA1y} = -\frac{1}{\mathrm{sin_DCA0}} \bullet \Delta\mathrm{cos_DCA1y}$$

(5) Compute the rotation of the velocity vector in the slant plane, ΔVDIR1y, due to the cross velocity component ΔVC1y. Parameter ΔVDIR1y is in radians.

$$\Delta\mathrm{VDIR1y} = \frac{1}{\mathrm{VM0}} \bullet \Delta\mathrm{VC1y} \qquad \Delta\mathrm{VDIR1y} > 0 \Leftrightarrow \text{CCW Rotation}$$

(6) Compute the rotation in the slant plane projection direction ΔANG1y due to the change in velocity direction and the change in the DCA.

$$\Delta\mathrm{ANG1y} = \Delta\mathrm{VDIR1y} + \mathrm{LOOK} \bullet \Delta\mathrm{DCA1y} \qquad \Delta\mathrm{ANG1y} > 0 \Leftrightarrow \text{CCW Rotation}$$

(7) Compute the change in the slant plane projection point, Δ**SPXY1y**, due to the change in image column coordinate, Δycol. Parameters ΔSPX1y and ΔSPY1y are in meters. Note: Small angle approximations are used for cosine and sine of ΔANG1y.

$$\Delta\mathrm{SPX1y} = \Delta\mathrm{ARP1y_SPX} - \Delta\mathrm{R1y}_{\mathrm{COA}}$$

$$\Delta\mathrm{SPY1y} = \Delta\mathrm{ARP1y_SPY} - \mathrm{R0}_{\mathrm{COA}} \bullet \Delta\mathrm{ANG1y}$$

Form the 2 x 2 sensitivity matrix $\mathbf{M}_{\mathrm{IL}}^{\mathrm{SPXY}}$ from the changes in slant plane projection points, (ΔSPX1x, ΔSPY1x) and (ΔSPX1y, ΔSPY1y) and the increments in the row coordinate and the column coordinate used, Δxrow and Δycol. Set sensitivity matrix $\mathbf{M}_{\mathrm{SPXY}}^{\mathrm{IL}}$ equal to the inverse of the $\mathbf{M}_{\mathrm{IL}}^{\mathrm{SPXY}}$.

$$\mathbf{M}_{\mathrm{IL}}^{\mathrm{SPXY}} = \begin{bmatrix} \dfrac{\Delta\mathrm{SPX1x}}{\Delta\mathrm{xrow}} & \dfrac{\Delta\mathrm{SPX1y}}{\Delta\mathrm{ycol}} \\ \dfrac{\Delta\mathrm{SPY1x}}{\Delta\mathrm{xrow}} & \dfrac{\Delta\mathrm{SPY1y}}{\Delta\mathrm{ycol}} \end{bmatrix} \qquad \mathbf{M}_{\mathrm{SPXY}}^{\mathrm{IL}} = \left(\mathbf{M}_{\mathrm{IL}}^{\mathrm{SPXY}}\right)^{-1}$$

Note: The matrix $\mathbf{M}_{\mathrm{IL}}^{\mathrm{SPXY}}$ is non-singular for any practical SAR imaging geometry.

11.4 Image To Scene Sensitivity Matrices

The sensitivity matrices that relate changes in image location and scene point position are listed in Table 11.3 above. The method for computing them is described below.

Scene Point Position ⇔ Image Location: $\mathbf{M}_{\mathrm{PT}}^{\mathrm{IL}}$ $\mathbf{M}_{\mathrm{GPXY}}^{\mathrm{IL}}$ $\mathbf{M}_{\mathrm{IL}}^{\mathrm{GPXY}}$

(1) Compute the 2 x 3 sensitivity matrix to compute the change in image grid location (Δ**IL**) due to a change in scene point position in ECF coordinates (Δ**PT**).

$$\mathbf{M}_{\mathrm{PT}}^{\mathrm{IL}} = \underbrace{\mathbf{M}_{\mathrm{SPXY}}^{\mathrm{IL}}}_{2\text{ x }2} \bullet \underbrace{\mathbf{M}_{\mathrm{PT}}^{\mathrm{SPXY}}}_{2\text{ x }3}$$

(2) Compute the 2 x 2 sensitivity matrix to compute the change in image grid location (Δ**IL**) due to a change in ground plane point position in ground plane coordinates (Δ**GPXY**). Also, compute the 2 x 2 sensitivity matrix to compute the change in ground plane point position (Δ**GPXY**) due to a change in image grid location (Δ**IL**).

$$\mathbf{M}_{\mathrm{GPXY}}^{\mathrm{IL}} = \underbrace{\mathbf{M}_{\mathrm{SPXY}}^{\mathrm{IL}}}_{2\text{ x }2} \bullet \underbrace{\mathbf{M}_{\mathrm{GPXY}}^{\mathrm{SPXY}}}_{2\text{ x }2} \qquad \mathbf{M}_{\mathrm{IL}}^{\mathrm{GPXY}} = \underbrace{\mathbf{M}_{\mathrm{SPXY}}^{\mathrm{GPXY}}}_{2\text{ x }2} \bullet \underbrace{\mathbf{M}_{\mathrm{IL}}^{\mathrm{SPXY}}}_{2\text{ x }2}$$

Scene Point HAE ⇔ Scene Point or Image Location: $\mathbf{M}_{\mathrm{HAE}}^{\mathrm{PT}}$ $\mathbf{M}_{\mathrm{HAE}}^{\mathrm{IL}}$

(1) Form the 3 x 1 sensitivity matrix/vector to compute the change in scene point position in ECF coordinates (Δ**PT**) due to a change in scene point HAE (ΔHAE). The image grid location is held constant and the scene point moves along the slant plane normal.

$$\mathrm{SF_SPZ} = \mathbf{uUP0} \bullet \mathbf{uSPZ} \qquad \mathbf{M}_{\mathrm{HAE}}^{\mathrm{PT}} = \frac{1}{\mathrm{SF_SPZ}} \bullet \mathbf{uSPZ}$$

(2) Form the 2 x 1 sensitivity matrix/vector to compute the change in image grid location in image coordinates (Δ**IL**) due to a change in scene point HAE (ΔHAE). The scene point latitude and longitude are held constant and the scene point moves normal to the surface of constant HAE (i.e. the point moves in the "up" direction from point **PT0**). The sensitivity vector is oriented in the "layover direction" in the image grid.

$$\mathbf{M}_{\mathrm{HAE}}^{\mathrm{IL}} = \underbrace{\mathbf{M}_{\mathrm{PT}}^{\mathrm{IL}}}_{2 \times 3} \bullet \mathbf{uUP0}$$

11.5 Adjustable Parameter Offset Sensitivity Matrices

The sensitivity matrices that relate changes in image grid location and scene point position due to changes in the Adjustable Parameter Offsets are listed in Table 11.4 and Table 11.5 above. For all matrices, the Adjustable Parameter Offsets are the independent parameters. For the matrices in Table 11.4, the changes in scene point are computed when the image grid location **IL0** is held constant. The method for computing them is described below.

Adjustable Parameter Offsets ➔ Slant Plane & Ground Plane Point
Image Grid Location Held Constant

$$\mathbf{MIL}_{\Delta\mathrm{ARP}}^{\mathrm{SPXY}} \quad \mathbf{MIL}_{\Delta\mathrm{VARP}}^{\mathrm{SPXY}} \quad \mathbf{MIL}_{\Delta\mathrm{R}}^{\mathrm{SPXY}} \quad \mathbf{MIL}_{\Delta\mathrm{ARP}}^{\mathrm{GPXY}} \quad \mathbf{MIL}_{\Delta\mathrm{VARP}}^{\mathrm{GPXY}} \quad \mathbf{MIL}_{\Delta\mathrm{R}}^{\mathrm{GPXY}}$$

(1) Form the 2 x 3 sensitivity matrix to compute the change in slant plane projection point (Δ**SPXY**) due to a change in ARP position adjustable parameter offset in ECF coordinates ($\Delta\mathbf{ARP}_{\mathrm{COA}}^{\mathrm{SCP}}$). Form the matrix from the slant plane unit vectors **uSPX** and **uSPY**.

$$\mathbf{MIL}_{\Delta\mathrm{ARP}}^{\mathrm{SPXY}} = \begin{bmatrix} \mathbf{uSPX}^{\mathrm{T}} \\ \mathbf{uSPY}^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} \mathrm{uSPX_X} & \mathrm{uSPX_Y} & \mathrm{uSPX_Z} \\ \mathrm{uSPY_X} & \mathrm{uSPY_Y} & \mathrm{uSPY_Z} \end{bmatrix}$$

(2) Compute the 2 x 3 sensitivity matrix to compute the change in slant plane projection point (Δ**SPXY**) due to a change in ARP velocity adjustable parameter offset in ECF coordinates ($\Delta\mathbf{VARP}_{\mathrm{COA}}$). The matrix is the sum of two matrices. The first matrix accounts for the change in ARP COA position due to a change in velocity offset. The second matrix accounts for the change in slant plane COA projection direction due to a change in velocity offset. See Section 11.3 and Figure 11.5 above.

$$\mathbf{MIL}_{\Delta\mathrm{VARP}}^{\mathrm{SPXY}} = \underbrace{\left(\mathrm{t0}_{\mathrm{COA}} - \mathrm{t}_{\mathrm{COA}}^{\mathrm{SCP}}\right) \bullet \begin{bmatrix} \mathbf{uSPX}^{\mathrm{T}} \\ \mathbf{uSPY}^{\mathrm{T}} \end{bmatrix}}_{\substack{\text{2 x 3 - Accounts For Change In} \\ \text{ARP COA Position}}} - \underbrace{\left(\frac{\mathrm{R0}_{\mathrm{COA}}}{\mathrm{VM0}}\right) \bullet \begin{bmatrix} 0 & 0 \\ \dfrac{\mathrm{LOOK}}{\mathrm{tan_DCA0}} & 1 \end{bmatrix} \bullet \begin{bmatrix} \mathbf{uVM}^{\mathrm{T}} \\ \mathbf{uVC}^{\mathrm{T}} \end{bmatrix}}_{\substack{\text{2 x 3 - Accounts For Change In Slant Plane} \\ \text{Projection Direction}}}$$

(3) Assign the sensitivity matrix/vector to compute the change in slant plane projection point (Δ**SPXY**) due to a change in COA range adjustable parameter offset (ΔR_BIAS).

$$\mathbf{MIL}_{\Delta R}^{SPXY} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

(4) Compute the 2 x 3 sensitivity matrix to compute the change in ground plane point (Δ**GPXY**) due to a change in ARP position adjustable parameter offset in ECF coordinates ($\Delta\mathbf{ARP}_{COA}^{SCP}$).

$$\mathbf{MIL}_{\Delta ARP}^{GPXY} = \mathbf{M}_{SPXY}^{GPXY} \bullet \mathbf{MIL}_{\Delta ARP}^{SPXY}$$

(5) Compute the 2 x 3 sensitivity matrix to compute the change in ground plane point (Δ**GPXY**) due to a change in ARP velocity adjustable parameter offset in ECF coordinates ($\Delta\mathbf{VARP}_{COA}$).

$$\mathbf{MIL}_{\Delta VARP}^{GPXY} = \mathbf{M}_{SPXY}^{GPXY} \bullet \mathbf{MIL}_{\Delta VARP}^{SPXY}$$

(6) Compute the 2 x 1 sensitivity matrix/vector to compute the change in ground plane point (Δ**GPXY**) due to a change in COA range adjustable parameter offset (ΔR_BIAS).

$$\mathbf{MIL}_{\Delta R}^{GPXY} = \mathbf{M}_{SPXY}^{GPXY} \bullet \mathbf{MIL}_{\Delta R}^{SPXY}$$

For the matrices in Table 11.5, the changes in slant plane projection point and image grid location are computed when the scene point position **PT0** is held constant. The method for computing them is described below.

Adjustable Parameter Offsets ➔ Slant Plane & Image Grid Location
Scene Point Position Held Constant

$$\mathbf{MPT}_{\Delta ARP}^{SPXY} \quad \mathbf{MPT}_{\Delta VARP}^{SPXY} \quad \mathbf{MPT}_{\Delta R}^{SPXY} \quad \mathbf{MPT}_{\Delta ARP}^{IL} \quad \mathbf{MPT}_{\Delta VARP}^{IL} \quad \mathbf{MPT}_{\Delta R}^{IL}$$

(1) Assign the sensitivity matrices to compute the change in slant plane projection point due to changes in the adjustable parameter offsets. The scene point position **PT0** is held constant.

$$\mathbf{MPT}_{\Delta ARP}^{SPXY} = -\mathbf{MIL}_{\Delta ARP}^{SPXY} \qquad \mathbf{MPT}_{\Delta VARP}^{SPXY} = -\mathbf{MIL}_{\Delta VARP}^{SPXY} \qquad \mathbf{MPT}_{\Delta R}^{SPXY} = \begin{bmatrix} +1 \\ 0 \end{bmatrix}$$

(2) Compute the 2 x 3 sensitivity matrix to compute the change in image grid location ($\Delta\mathbf{IL}$) due to a change in ARP position adjustable parameter offset in ECF coordinates ($\Delta\mathbf{ARP}_{\mathrm{COA}}^{\mathrm{SCP}}$). The scene point position **PT0** is held constant.

$$\mathbf{MPT}_{\Delta\mathrm{ARP}}^{\mathrm{IL}} = \mathbf{M}_{\mathrm{SPXY}}^{\mathrm{IL}} \bullet \mathbf{MPT}_{\Delta\mathrm{ARP}}^{\mathrm{SPXY}}$$

(3) Compute the 2 x 3 sensitivity matrix to compute the change in image grid location ($\Delta\mathbf{IL}$) due to a change in ARP velocity adjustable parameter offset in ECF coordinates ($\Delta\mathbf{VARP}_{\mathrm{COA}}$). The scene point position **PT0** is held constant.

$$\mathbf{MPT}_{\Delta\mathrm{VARP}}^{\mathrm{IL}} = \mathbf{M}_{\mathrm{SPXY}}^{\mathrm{IL}} \bullet \mathbf{MPT}_{\Delta\mathrm{VARP}}^{\mathrm{SPXY}}$$

(4) Compute the 2 x 1 sensitivity matrix/vector to compute the change in image grid location ($\Delta\mathbf{IL}$) due to a change in COA range adjustable parameter offset (ΔR_BIAS). The scene point position **PT0** is held constant.

$$\mathbf{MPT}_{\Delta\mathrm{R}}^{\mathrm{IL}} = \mathbf{M}_{\mathrm{SPXY}}^{\mathrm{IL}} \bullet \mathbf{MPT}_{\Delta\mathrm{R}}^{\mathrm{SPXY}}$$

12 Projection Error Propagation

The process of mapping image grid locations to geolocated points in the imaged scene is a fundamental step in the exploitation of SAR complex image products. For most complex images, the image pixel grid is a distorted representation of the imaged scene. Projection and resampling to a ground plane pixel grid is needed to enable even the most basic image exploitation. For a given application, the absolute accuracy of the projection may or may not be of concern. For any image product, optional error parameters may be included that enable the expected accuracy of the image to scene and scene to image projections to be computed. For SAR sensors and processing systems designed for precise geolocation measurements, the optional parameters may be provided with every image product. The parameters that may be provided are contained in the optional Error Statistics parameter block. The Error Statistics parameter block is defined in Table 3-12 of the SICD Volume 1 Design & Implementation Description Document.

The Error Statistics block includes estimates of the error statistics for the primary sources of projection error for a typical SAR sensor and image formation processor. For any image, the fundamental projection computation is the mapping from image grid location to an R/Rdot contour. For a given image grid location, the R/Rdot contour is defined by the COA projection set. See Section 4. Consider image grid location **IL** and its COA projection set.

$$\text{Image Location } \mathbf{IL} = \begin{bmatrix} \text{xrow} \\ \text{ycol} \end{bmatrix} \qquad \text{Projection Set: } \begin{Bmatrix} t_{COA} & \mathbf{ARP}_{COA} & R_{COA} \\ t_{COA}^{SCP} & \mathbf{VARP}_{COA} & \text{Rdot}_{COA} \end{Bmatrix}$$

For grid location **IL**, an error in the computed R/Rdot contour is due to an error in one or more of the parameters of the projection set. Errors in the computed ARP COA position and/or velocity contribute directly. Errors in the radar sensor parameters (e.g. internal signal path delay times) lead to errors in computing range from signal delay time. Errors in the propagation model (e.g. an incorrect tropospheric delay estimate) also lead to errors in computing range from signal propagation time.

Consider an image location/scene point projection pair **IL** and **PT**. The image projection functions that connect the projection pair are shown in Figure 12-1. See Section 11 for the description of an image/scene projection pair. For a given projection pair, errors in the collection and/or processing will result in errors in both the image to scene and the scene to image projections. Errors in the scene surface model will result in errors in the computed scene point for a given image location. The following sections describe the impact of both types of errors on the image to scene and scene to image projections. The errors associated with the SAR image are referred to as the “image” errors. The image errors are the result of errors in the collection and/or processing of the complex image product. The image errors lead to errors in the COA projection set computed for a given image grid location. The effect of an error in the scene surface model is referred to as “surface model” error. The effect of an error in scene ground height for a locally level scene surface is described in detail.

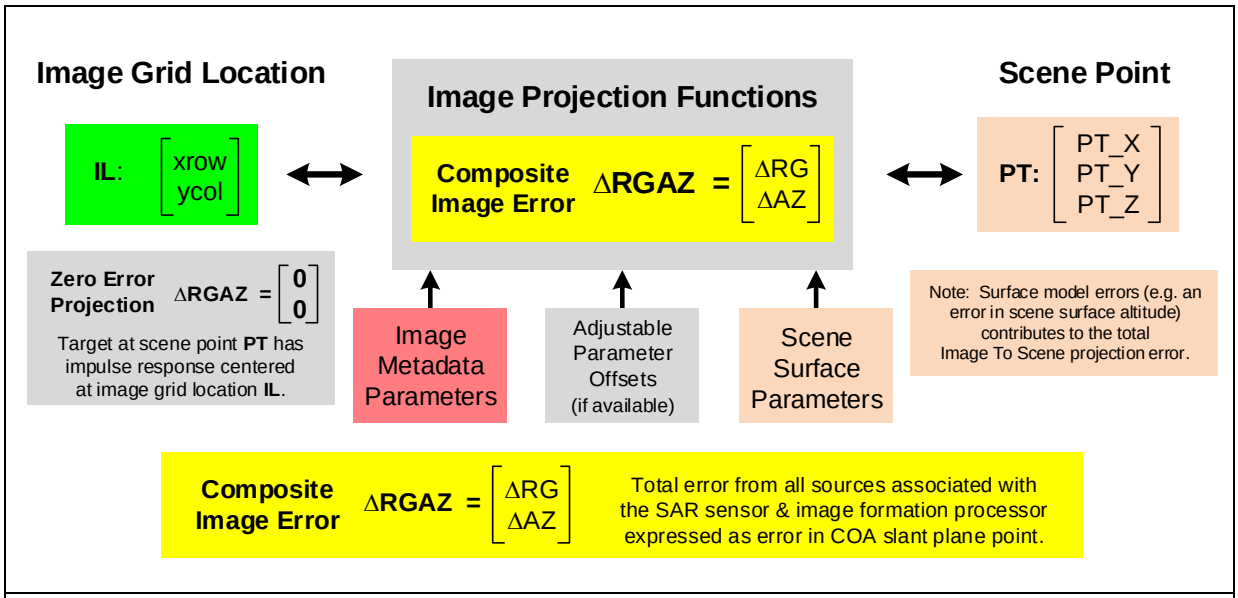


Figure 12-1 Image Projection Pair IL & PT
Projection error due to composite image error and/or surface model error.

Error Propagation Computation & Adjustable Parameter Offsets

Consider the image location/scene point projection pair **IL** and **PT** shown in Figure 12-1. The image projection functions allow for the use of Adjustable Parameter Offsets for both the image to scene and scene to image projections. The goal of the adjustable parameter offsets is to compensate for the projection errors that would occur using only the metadata provided with the image. The adjustable parameter offsets were chosen to match the primary sources of image projection error for SAR systems. See Section 1.3 and Table 1-3. For a given image product, the error statistics included in the product are presumed to be valid for projections computed using only the metadata included in the image or, equivalently, for the adjustable parameter offsets set equal to zero. In following sections, we assume that all image location/scene point projection pairs are computed with all adjustable parameter offsets set equal to zero.

Zero Error Projection

Consider the image location/scene point projection pair **IL0** and **PT0** shown in Figure 12-2. The example shown is referred to as the "zero error" projection in that all sources of error are zero. All sources of image error (ARP position vs. time, etc.) are zero. For the example shown, the imaged scene is a locally level area. The scene surface model is a ground plane with zero surface model error. The pixel grid near **IL0** is shown in the left side of the figure. The ground plane and scene point **PT0** are shown in right side of the figure. Also, shown is the COA slant plane for point **PT0**. See Section 11.2 for a description of COA slant plane. For zero error projection, an ideal point scatterer located at scene point **PT0**, will have its image signal response centered at image grid location **IL0**. Shown in the left side of the figure are image grid locations near **IL0** but displaced in the xrow coordinate (blue) or the ycol coordinate (red). Shown in the right side of the figure are the projection contours for

these image grid locations as they pass through the COA slant plane and scene surface/ground plane near point **PT0**. The projection contours for image grid locations near **IL0** are may be approximated by straight lines as they pass near scene point **PT0.** The straight line approximations are oriented normal to **PT0** COA slant plane.

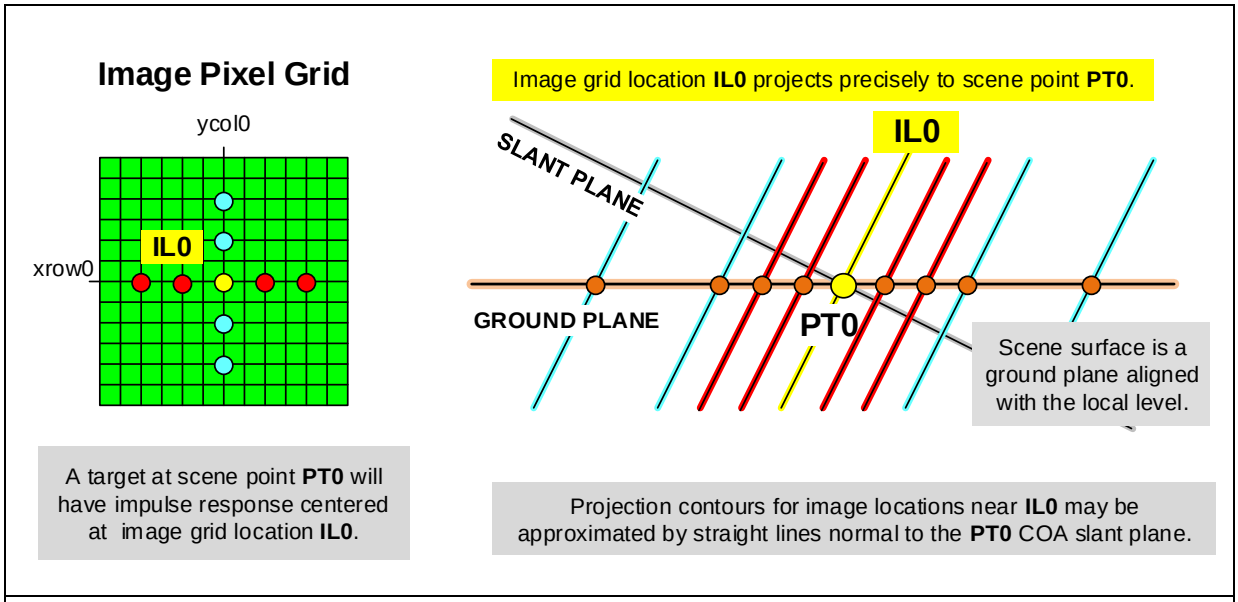


Figure 12-2 Zero Error Projection

Example projection geometry with zero composite image error and zero surface model error.

Error Projection w/ Image Errors

Consider the image location/scene point projections example shown in Figure 12-3. For the example shown, the projections are in error due to errors in the image metadata. Similar to the example in Figure 12-2, the imaged scene is a locally level area and the scene surface model is a ground plane with zero surface model error. For an ideal point scatterer at scene point **PT0**, the image signal response is centered at image location **IL0**. Given true image location **IL0**, the image to scene (I2S) projection yields scene point **E:PT0** equal to an "estimate" of the scene point position. Given the true scene point position **PT0**, the scene to image (S2I) projection yields image location **E:IL0** equal to an "estimate" of the image location. For the I2S projection, the error in estimated scene point is **ΔPT0**. For the S2I location, the error in estimated image location is **ΔIL0**.

Image To Scene Projection Error:

$$\underbrace{\Delta\mathbf{PT0}}_{\text{ERROR}} = \underbrace{\mathbf{PT0}}_{\text{TRUE VALUE}} - \underbrace{\mathbf{E:PT0}}_{\text{ESTIMATED VALUE}}$$

Scene To Image Projection Error:

$$\underbrace{\Delta\mathbf{IL0}}_{\text{ERROR}} = \underbrace{\mathbf{IL0}}_{\text{TRUE VALUE}} - \underbrace{\mathbf{E:IL0}}_{\text{ESTIMATED VALUE}}$$

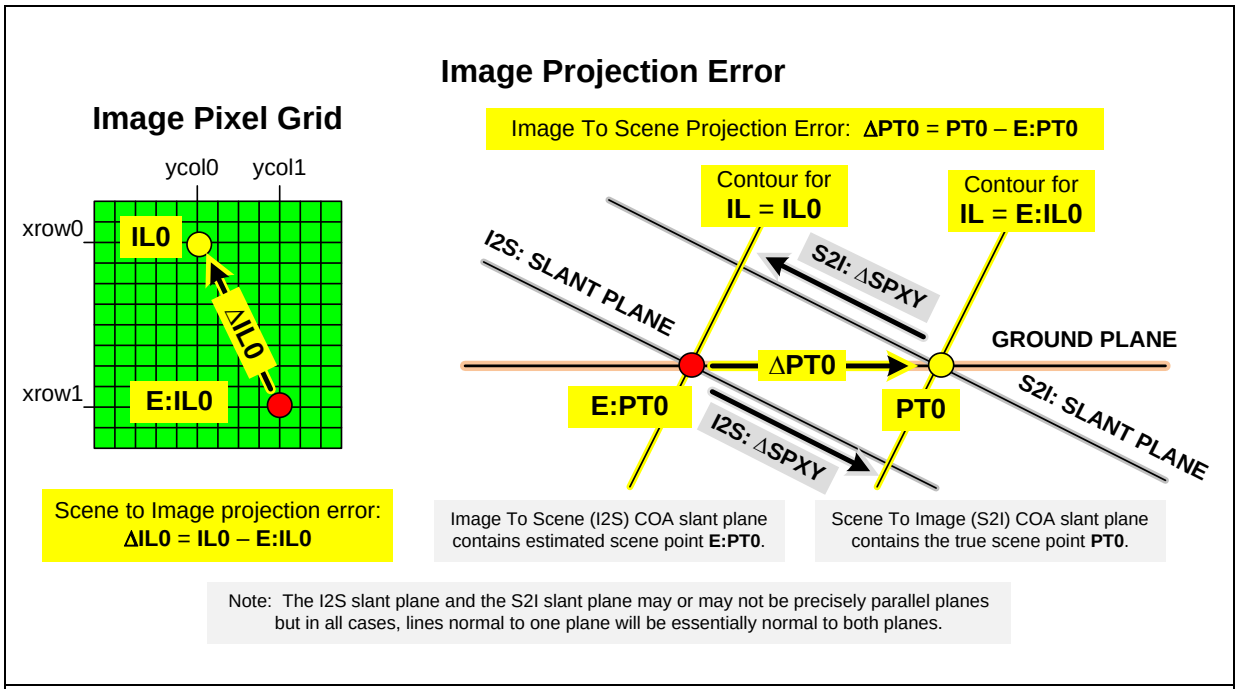


Figure 12-3 Projection With Image Error

Example projection with error due to image error alone (i.e. zero surface model error).

For the I2S projection shown in Figure 12-3, true image location **IL0** and estimated scene point **E:PT0** are a projection pair. For the **IL0**/**E:PT0** pair, we can compute the I2S slant plane coordinates SPX and SPY for which **E:PT0** is located at the origin. See Section 11.2. For true scene point **PT0**, the true projection contour intersects the I2S slant plane at **I2S: ΔSPXY** = I2S: (ΔSPXI, ΔSPYI). For the S2I projection, estimated image location **E:IL0** and true scene point **PT0** are a projection pair. For the **E:IL0**/**PT0** pair, we can compute the S2I slant plane coordinates SPX and SPY for which **PT0** is located at the origin. For true image location **IL0**, the projection contour intersects the S2I slant plane at **S2I: ΔSPXY** = S2I: (ΔSPXS, ΔSPYS). As indicated in Figure 12-3, the slant plane displacements due to the image error are opposite in sign and approximately equal in magnitude.

$$\mathbf{I2S:\Delta SPXY} \approx -\mathbf{S2I:\Delta SPXY} \qquad \mathbf{I2S}:\begin{bmatrix}\Delta\mathrm{SPXI}\\ \Delta\mathrm{SPYI}\end{bmatrix} \approx -\mathbf{S2I}:\begin{bmatrix}\Delta\mathrm{SPXS}\\ \Delta\mathrm{SPYS}\end{bmatrix}$$

For a given image projection pair, the combined effect of all sources of image projection error is to displace the "correct" projection contour relative to the "computed" projection contour. The displacement is relative to the COA slant plane origin. For the I2S projection, the correct contour (through true scene point PT0) is displaced by **I2S:ΔSPXY** from the I2S estimated scene point **E:PT0**. For the S2I projection, the correct contour (for true image location **IL0**) is displaced by **S2I:ΔSPXY** from the S2I scene point **PT0**.

Composite Image Error

The combined effect of all sources of image projection error is referred as the composite image error. For a given image location, the composite error is effectively an error in computed COA range and range rate. For a given image grid location **IL**, the composite error is $\Delta\mathbf{RGAZ} = (\Delta RG, \Delta AZ)^T$ where, for convenience, the error in range rate is expressed as an error in azimuth or cross range in the slant plane. The composite errors ΔRG andΔ AZ are expressed in meters and are defined as follows.

For image grid location **IL**, the COA projection set is computed. The ARP COA position and velocity are accepted as truth. The computed range and range rate are treated as estimates, $E{:}R_{COA}$ and $E{:}Rdot_{COA}$.

$$\text{Image Location } \mathbf{IL} = \begin{bmatrix} xrow \\ ycol \end{bmatrix} \qquad \text{Projection Set: } \begin{Bmatrix} t_{COA} & \mathbf{ARP}_{COA} & E{:}R_{COA} \\ t_{COA}^{SCP} & \mathbf{VARP}_{COA} & E{:}Rdot_{COA} \end{Bmatrix}$$

Scene point **PT** is a point in the imaged scene with image response centered at image location **IL**. The range and range rate to **PT** relative the ARP COA position and velocity, R_PT_{COA} and $Rdot_PT_{COA}$, are accepted as the true range and range rate. The composite range error, ΔRG, is defined as follows.

$$\text{Composite Range Error at } \mathbf{IL}: \qquad \underbrace{\Delta RG}_{\substack{\text{RANGE}\\\text{ERROR}}} = \underbrace{R_PT_{COA}}_{\substack{\text{TRUE}\\\text{RANGE}}} - \underbrace{E{:}R_{COA}}_{\substack{\text{ESTIMATED}\\\text{RANGE}}}$$

The composite azimuth error is defined as follows by converting composite error in range rate, ΔRDOT, to a displacement in azimuth, ΔAZ.

$$\text{Composite Range Rate Error:} \qquad \underbrace{\Delta RDOT}_{\substack{\text{RANGE RATE}\\\text{ERROR}}} = \underbrace{Rdot_PT_{COA}}_{\substack{\text{TRUE}\\\text{RANGE RATE}}} - \underbrace{E{:}Rdot_{COA}}_{\substack{\text{ESTIMATED}\\\text{RANGE RATE}}}$$

$$\text{Composite Azimuth Error:} \qquad \underbrace{\Delta AZ}_{\substack{\text{AZIMUTH}\\\text{ERROR}}} = \underbrace{LOOK \bullet \frac{E{:}R_{COA}}{VM \bullet \sin\left(DCA_{COA}\right)}}_{\Delta RDOT \text{ to } \Delta AZ \text{ Scale Factor}} \bullet \Delta RDOT$$

The scale factor that translates range rate error (ΔRDOT) to azimuth error has units of seconds and is side of track (i.e. LOOK) dependent. The choice to express the range rate error in terms of an azimuth displacement allows the errors to be expressed in meters for both range and azimuth. Also, the choice of azimuth error allows direct conversion to slant plane displacements **I2S:ΔSPXY** and **S2I:ΔSPXY**. Shown in Figure 12-4 are the slant plane displacements for a given value of composite error. Shown in left side of the figure is the displacement for the Image To Scene projection. Shown in the right side of the figure is he displacement for the Scene To Image projection. A commonly used pixel grid is the slant plane (rg,az) grid (i.e. Grid Type RGAZIM). Shown in Figure 12-5 is an example slant plane grid with a non-zero composite error. The true image grid location **IL0** is the center of the image signal response for a scatterer at scene point **PT0**. The effect of the composite RGAZ error may be observed directly in the range, azimuth image grid: **ΔIL0 = - RGAZ**.

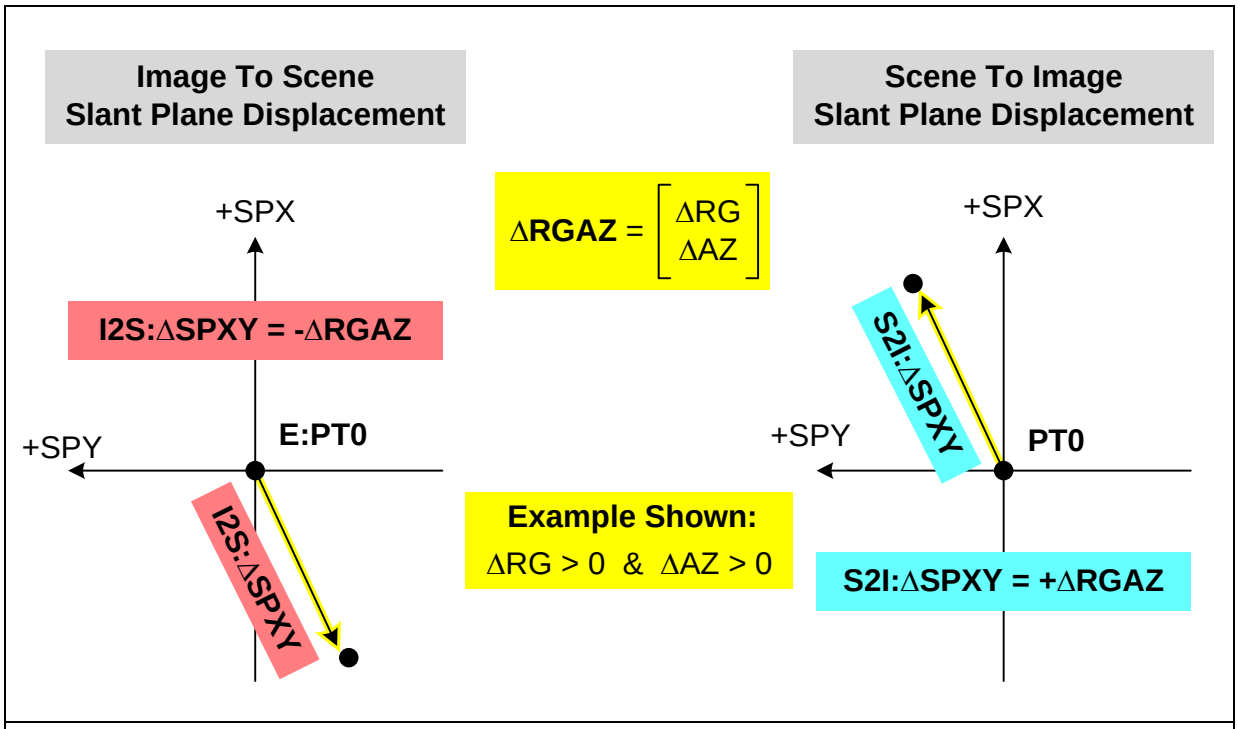


Figure 12-4 Displacement Due To Composite Error

Example I2S & S2I slant plane displacement due to a composite image error ΔRGAZ.

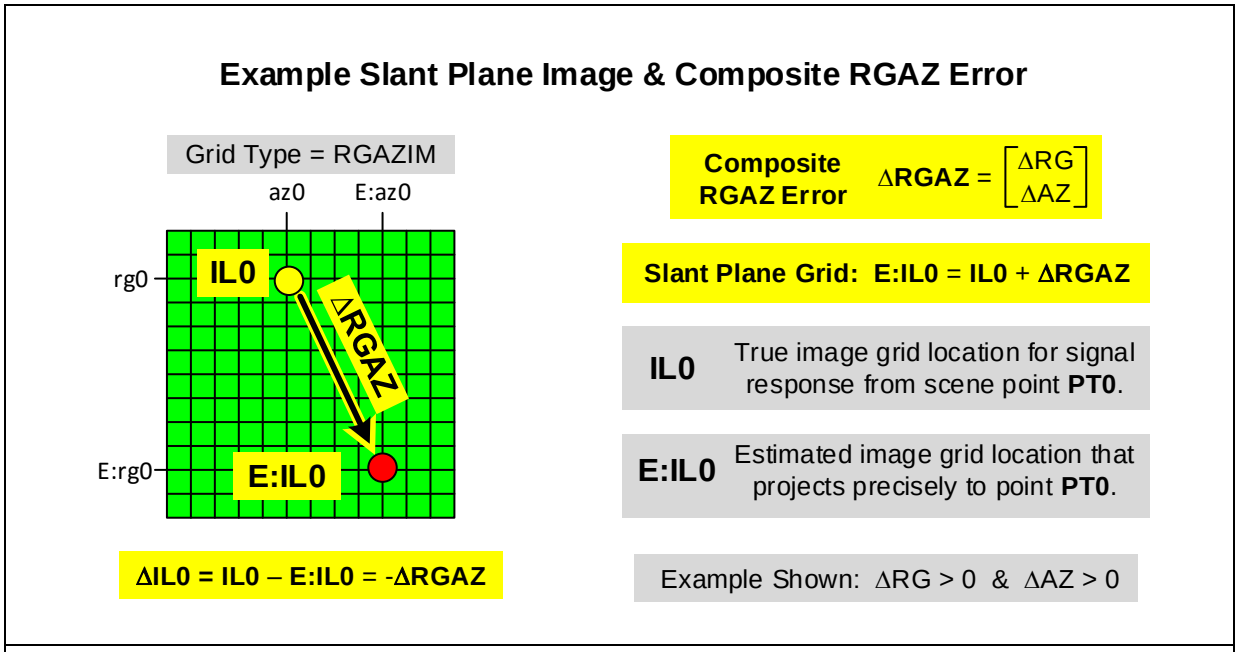


Figure 12-5 Example Slant Plane Image Grid

The composite RGAZ error is equal in magnitude but opposite in sign of S2I image error .

Error Propagation Computations

The error propagation computations in the following sections may be applied to any image product. The computations are presented for scene point **PT0** and image grid location **IL0** as shown in the examples above. For an ideal point scatterer at true scene point **PT0**, the image signal response is centered at true image grid location **IL0**. The error propagation description is provided for both the Image to Scene and the Scene to Image projections.

For each projection considered, the effect of a given error source on the parameter being computed is provided. Also provided is the relationship between the statistics of the error source and the statistics of the error in the parameter being computed. For all parameters, the errors are defined using the following convention. For a given parameter with true value X and estimated value E:X, the error ΔX is defined as follows.

Error Convention For All Parameters: $$\underbrace{\Delta X}_{\text{ERROR}} = \underbrace{X}_{\substack{\text{TRUE}\\\text{VALUE}}} - \underbrace{E{:}X}_{\substack{\text{ESTIMATED}\\\text{VALUE}}}$$

The image to scene projection of true image location **IL0** yields estimated scene point position **E:PT0**. The error in the estimated position is Δ**PT0**. The scene to image projection of true scene point position **PT0** yields estimated image location **E:IL0.** The error in estimated image location is Δ**IL0**.

Image To Scene Projection Error: $$\underbrace{\Delta\mathbf{PT0}}_{\text{ERROR}} = \underbrace{\mathbf{PT0}}_{\substack{\text{TRUE}\\\text{VALUE}}} - \underbrace{\mathbf{E:PT0}}_{\substack{\text{ESTIMATED}\\\text{VALUE}}}$$

Scene To Image Projection Error: $$\underbrace{\Delta\mathbf{IL0}}_{\text{ERROR}} = \underbrace{\mathbf{IL0}}_{\substack{\text{TRUE}\\\text{VALUE}}} - \underbrace{\mathbf{E:IL0}}_{\substack{\text{ESTIMATED}\\\text{VALUE}}}$$

In following description, the effects of the errors associated with the SAR image are discussed in detail. For a given image/scene projection pair, the image error may be expressed as a composite RGAZ error. The composite error is the sum of the effects of all components of the error. For a given source of image error, e.g. an error is COA position or velocity, the effect is dependent on the specific projection geometry of the image/scene point pair. The method for computing the contribution to composite RGAZ error is provided for the primary sources of projection error. The effect of the composite RGAZ error on both image to scene and scene to image projections is provided.

The projection errors that arise due to ARP position and velocity errors are computed for the following idealized error in the estimated ARP position versus time. The error is modelled as a simple linear error in position versus time in ECF coordinates. The position error is the ARP position error at SCP COA time. The velocity error is assumed to be constant over the collection time for the phase history data used to form the image. An example ARP position error versus time is shown in Figure 12-6. For the image grid shown, the COA time varies across the image pixel grid. The effect of the velocity error on the COA position varies with COA time of the image pixel array.

$$\underbrace{\Delta\mathbf{ARP}(t)}_{\substack{\text{ARP ERROR}\\ \text{VS TIME}}} = \underbrace{\mathbf{ARP}(t)}_{\substack{\text{TRUE ARP}\\ \text{TRAJECTORY}\\ \text{VS TIME}}} - \underbrace{\mathbf{E:ARP}(t)}_{\substack{\text{ESTIMATED ARP}\\ \text{TRAJECTORY}\\ \text{VS TIME}}} \qquad \Delta\mathbf{ARP}(t) = \underbrace{\Delta\mathbf{ARP}_{COA}^{SCP} + \Delta\mathbf{VARP}\bullet\left(t - t_{COA}^{SCP}\right)}_{\substack{\text{LINEAR ERROR}\\ \text{VS TIME}}}$$

$$\underbrace{\Delta\mathbf{ARP}_{COA}^{SCP} = \begin{bmatrix}\Delta ARP_SCP_X\\ \Delta ARP_SCP_Y\\ \Delta ARP_SCP_Z\end{bmatrix}}_{\substack{\text{Error in ARP Position at SCP COA time}\\ \text{in ECF Coordinates}}} \qquad \underbrace{\Delta\mathbf{VARP} = \begin{bmatrix}\Delta VARP_X\\ \Delta VARP_Y\\ \Delta VARP_Z\end{bmatrix}}_{\substack{\text{Error in ARP Velocity}\\ \text{in ECF Coordinates}}}$$

The ARP trajectory error model used for computing errors in the image projection functions is matched to the model used for specifying the Adjustable Parameter Offsets. See Section 1-3 and Table 1-3. The common model for ARP trajectory error has the following utility. For a given pair of ARP position and velocity error values, the computed projection errors are precisely equal to the errors that will be compensated using the same values for the adjustable parameter offsets.

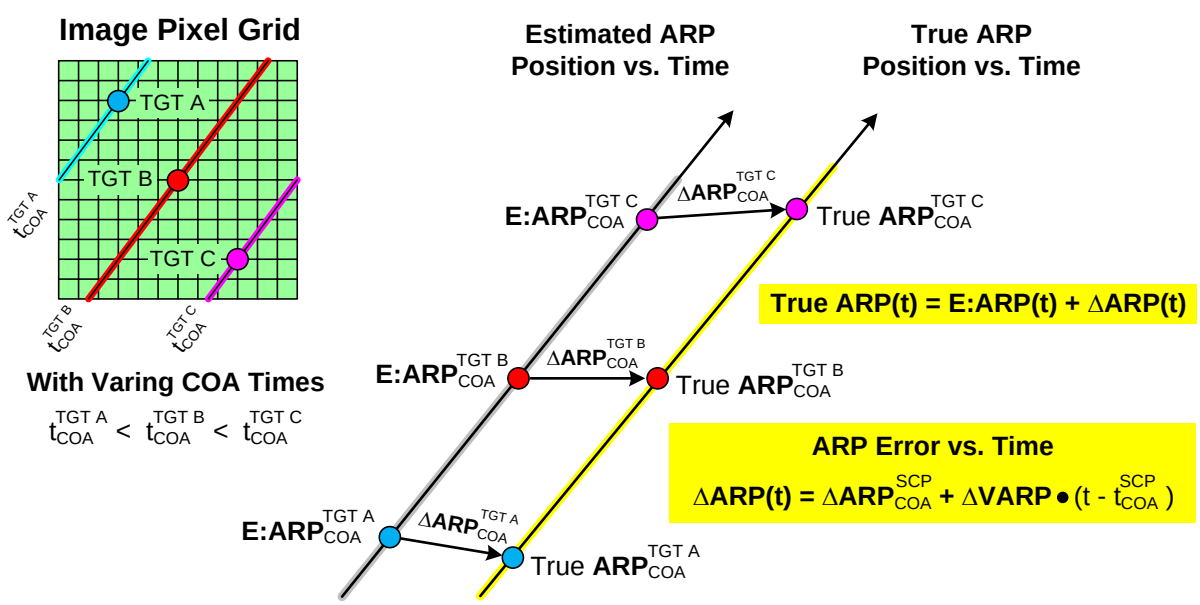


Figure 12-6 ARP Position & Velocity Error Model

The ARP error versus time is modelled as on offset at the SCP COA plus a constant velocity error.

The effect of an error in the scene surface model is presented for a locally level scene. The projection error is computed for a scene surface that is at true height HAE0 above the ellipsoid. The surface model estimated height is E:HAE0 and the error is ΔHAE.

Scene Height Error: $\underbrace{\Delta HAE}_{\text{ERROR}} = \underbrace{HAE0}_{\substack{\text{TRUE}\\ \text{HEIGHT}}} - \underbrace{E{:}HAE0}_{\substack{\text{ESTIMATED}\\ \text{HEIGHT}}}$

For more complex surface models, the error in the surface model may be of many forms. The various errors that may occur in the surface model are not addressed. For any surface model, the Image to Scene and Scene to Image projection functions may be used to compute the effect of a given surface model error.

12.1 Projection Geometry Parameters

(1) For the image/scene projection pair, compute the projection geometry parameters described in Section 11.1. For an image to scene projection pair, **IL0/E:PT0**, compute the COA projection set for image location **IL0**. For a scene to image projection, compute the COA projection set for the estimated image location **E:IL0**.

For Image To Scene: **IL0** ➔ Projection Set: $\left\{ \begin{matrix} t_{COA} & \mathbf{E:ARP}_{COA} & E{:}R_{COA} \\ t_{COA}^{SCP} & \mathbf{E:VARP}_{COA} & E{:}Rdot_{COA} \end{matrix} \right\}$

For Scene To Image: **E:IL0** ➔ Projection Set: $\left\{ \begin{matrix} t_{COA} & \mathbf{E:ARP}_{COA} & E{:}R_{COA} \\ t_{COA}^{SCP} & \mathbf{E:VARP}_{COA} & E{:}Rdot_{COA} \end{matrix} \right\}$

For both projections, the projection set is computed from the image metadata. All Adjustable Parameter Offsets are set equal to zero.

(2) Compute unit normal to the scene surface model and the unit UP vector. For an image to scene projection, compute the unit vectors at the estimated scene point **E:PT0**. For a scene to image projection, compute the unit vectors at scene point **PT0**. See Figure 11-2.

uGPN0	Unit normal to the scene surface at the scene point.
uUP0	Unit normal to the surface of constant HAE at the scene point.

(3) Compute the parameters that define the Ground Plane Coordinates (GPC) and the Slant Plane Coordinates (SPC) with origin at the scene point. For an image to scene projection, the origin is at scene point **E:PT0**. For a scene to image projection, the origin is at scene point **PT0**. Compute the GPC basis vectors and SPC basis vectors. All vectors are in ECF coordinates. Also, compute the cosine and sine of the angles GRAZ, TWST and DCA that describe the COA geometry.

GP Coordinates Basis Vectors in ECF: { **uGPX**, **uGPY**, **uGPZ** }

SP Coordinates Basis Vectors in ECF: { **uSPX**, **uSPY**, **uSPZ** }

GRAZ: Grazing angle TWST: Twist angle DCA: Doppler Cone Angle

12.2 Projection Sensitivity Parameters

(1) Compute the sensitivity matrices that relate changes in image location, **ΔIL**, slant plane projection point, **ΔSPXY**, ground plane point position, **ΔGPXY**, and scene point position in ECF coordinates, **ΔPT**. See Table 11-1. For an I2S projection, the changes are relative to image location **IL0** and scene point **E:PT0**. For a S2I projection, the changes are relative to **E:IL0** and scene point **PT0**.

$$\underbrace{\Delta\mathbf{IL} = \begin{bmatrix} \Delta\text{xrow} \\ \Delta\text{ycol} \end{bmatrix}}_{\text{Change In Image Location}} \quad \underbrace{\Delta\mathbf{SPXY} = \begin{bmatrix} \Delta\text{SPX} \\ \Delta\text{SPY} \end{bmatrix}}_{\substack{\text{Change In Slant Plane XY} \\ \text{Projection Point}}} \quad \underbrace{\Delta\mathbf{GPXY} = \begin{bmatrix} \Delta\text{GPX} \\ \Delta\text{GPY} \end{bmatrix}}_{\substack{\text{Change In Ground Plane Point} \\ \text{Specified In GP Coordinates}}} \quad \underbrace{\Delta\mathbf{PT} = \begin{bmatrix} \Delta\text{PT_X} \\ \Delta\text{PT_Y} \\ \Delta\text{PT_Z} \end{bmatrix}}_{\substack{\text{Change in Scene Point} \\ \text{ECF Coordinates}}}$$

(1.1) Compute the sensitivity matrices defined in Table 11-2 and listed below. See Section 11.3 for the details of the computations.

Scene Point ⇔ Slant Plane Projection Point: $\mathbf{M}_{\text{PT}}^{\text{SPXY}}$ $\mathbf{M}_{\text{GPXY}}^{\text{PT}}$ $\mathbf{M}_{\text{GPXY}}^{\text{SPXY}}$ $\mathbf{M}_{\text{SPXY}}^{\text{GPXY}}$

Image Location ⇔ Slant Plane Projection Point: $\mathbf{M}_{\text{IL}}^{\text{SPXY}}$ $\mathbf{M}_{\text{SPXY}}^{\text{IL}}$

(1.2) Compute the sensitivity matrices defined in Table 11-3 and listed below. See Section 11.4 for the details of the computations.

Scene Point Position ⇔ Image Location: $\mathbf{M}_{\text{PT}}^{\text{IL}}$ $\mathbf{M}_{\text{GPXY}}^{\text{IL}}$ $\mathbf{M}_{\text{IL}}^{\text{GPXY}}$

Scene Point HAE ⇔ Scene Point or Image Location: $\mathbf{M}_{\text{HAE}}^{\text{PT}}$ $\mathbf{M}_{\text{HAE}}^{\text{IL}}$

(2) The ARP position versus time error model is the same model used for the ARP adjustable parameter offsets. The projection sensitivity to position and velocity error terms is identical to the projection sensitivity to ARP adjustable parameter offsets. Compute the following sensitivity matrices defined in Table 11-4 and Table 11-5. See Section 11.5 for the details of the computations.

Scene to Image - Scene Point Held Constant: $\mathbf{MPT}_{\Delta\text{ARP}}^{\text{SPXY}}$ $\mathbf{MPT}_{\Delta\text{VARP}}^{\text{SPXY}}$

Assign the matrices that translate ARP positon and velocity errors into composite RGAZ errors.

ARP Position Error at SCP COA Time ➔ RGAZ Error: $\mathbf{M}_{\Delta\text{ARP}}^{\text{RGAZ}} = \mathbf{MPT}_{\Delta\text{ARP}}^{\text{SPXY}}$

ARP Velocity Error ➔ RGAZ Error: $\mathbf{M}_{\Delta\text{VARP}}^{\text{RGAZ}} = \mathbf{MPT}_{\Delta\text{VARP}}^{\text{SPXY}}$

(3) The composite RGAZ error due to a given realization of ARP position and velocity errors, $\Delta\mathbf{ARP}_{\mathrm{COA}}^{\mathrm{SCP}}$ and $\Delta\mathbf{VARP}$, is computed as shown.

$$\Delta\mathbf{RGAZ}_{\mathrm{ARP_PV}} = \underbrace{\mathbf{M}_{\Delta\mathrm{ARP}}^{\mathrm{RGAZ}}}_{2\mathrm{x}3} \bullet \underbrace{\Delta\mathbf{ARP}_{\mathrm{COA}}^{\mathrm{SCP}}}_{\substack{\text{ARP Position}\\ \text{Error In ECF}}} + \underbrace{\mathbf{M}_{\Delta\mathrm{VARP}}^{\mathrm{RGAZ}}}_{2\mathrm{x}3} \bullet \underbrace{\Delta\mathbf{VARP}}_{\substack{\text{ARP Velocity}\\ \text{Error In ECF}}}$$

Form the combined 2 x 6 matrix $\mathbf{M}_{\mathrm{ARP_PV}}^{\mathrm{RGAZ}}$ for use in computing the 2 x 2 covariance matrix for the RGAZ error from a 6 x 6 combined position and velocity covariance matrix. Note: The position and velocity errors in the expression above are in ECF coordinates.

$$\mathbf{M}_{\mathrm{ARP_PV}}^{\mathrm{RGAZ}} = \underbrace{\begin{bmatrix} \mathbf{MPT}_{\Delta\mathrm{ARP}}^{\mathrm{SPXY}} & \mathbf{MPT}_{\Delta\mathrm{VARP}}^{\mathrm{SPXY}} \end{bmatrix}}_{2\ \mathrm{x}\ 6}$$

(4) A given RGAZ error displaces the correct or true projection contour relative to the computed projection contour. See Figures 12-3 and 12-4. For a scene to image projection, the slant plane displacement is equal to the RGAZ error. For an image to scene projection, the slant plane displacement equal to the negative of the RGAZ error. Assign the following 2 x 2 matrices for convenience in translating a $\Delta\mathbf{RGAZ}$ error to a slant plane displacement $\Delta\mathbf{SPXY}$.

Scene To Image Projection, $\Delta\mathbf{RGAZ}$ to $\Delta\mathbf{SPXY}$: $\mathbf{MPT}_{\mathrm{RGAZ}}^{\mathrm{SPXY}} = \begin{bmatrix} +1 & 0 \\ 0 & +1 \end{bmatrix}$

Image To Scene Projection, $\Delta\mathbf{RGAZ}$ to $\Delta\mathbf{SPXY}$: $\mathbf{MIL}_{\mathrm{RGAZ}}^{\mathrm{SPXY}} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

12.3 Composite RGAZ Error

For a given image, the projection error due to errors in the collection and/or processing will vary across the image and scene as the projection geometry varies. For a given image/scene projection pair, the combined effects of all error sources may be expressed as a composite RGAZ error. The composite error is computed based upon the specific COA projection geometry. The method for computing the contribution to the composite error for the primary error components is described in detail. Also described is the method for computing the composite error statistics as a function of the expected error statistics of the individual components.

The sources of error in the image collection and/or processing that produce errors in the image projections include (1) ARP position and velocity error, (2) radar sensor range bias error and timing clock error, and (3) propagation delay estimation errors. The contribution to composite RGAZ error is computed for the following error components.

$\Delta\mathbf{ARP}_{COA}^{SCP}$ ARP position error at SCP COA time in ECF coordinates (meters).

$\Delta\mathbf{VARP}$ ARP velocity error in ECF coordinates (meters/sec). Assumed constant during collection period for the data used to form the image.

ΔR_BIAS Range bias in the computed COA range that is constant across the image (meters). The true range is equal to the estimated range plus the range bias: $R_{COA} = E{:}R_{COA} + \Delta R_BIAS$.

CLK_SF Radar timing clock error expressed as a scale factor. The clock frequency error Δf_CLK is equal to the scale factor times the estimated value clock frequency: $\Delta f_CLK = f_CLK_{EST} \bullet CLK_SF$. The true clock frequency is equal to the estimated frequency plus the error: $f_CLK = f_CLK_{EST} + \Delta f_CLK$.

ΔR_TROP Range error due to an error in the estimated troposphere delay expressed as a range error (meters). The true range is equal to the estimated range plus the range error: $R_{COA} = E{:}R_{COA} + \Delta R_TROP$.

ΔR_IONO Range error due to an error in estimated ionosphere delay expressed as a range error (meters). The true range is equal to the estimated range plus the range error: $R_{COA} = E{:}R_{COA} + \Delta R_IONO$.

The radar timing clock controls the timing for both “fast time” and “slow time”. The effect of an error in the radar timing clock is a range error that is proportional to the measured range and a range rate error that is proportional to measured range rate. For a given transmitted pulse, the range is derived from the time of arrival based upon the number of timing clock periods. If the true clock frequency is higher than the value used in the image formation processing, the estimated range will be larger than the true range. The estimated range rate will also be larger in magnitude than the true range rate. For a given value of clock scale factor error, the error in measured range and range rate is ΔR_CLK and ΔRDOT_CLK.

$$\Delta R_CLK = -E{:}R_{COA} \bullet CLK_SF \qquad \Delta RDOT_CLK = -E{:}Rdot_{COA} \bullet CLK_SF$$

The error in clock scale factor can be expressed as azimuth error by combining the expressions for $Rdot_{COA}$ and the scale factor that scales a range rate error to an azimuth error.

$$\Delta AZ_CLK = LOOK \bullet E{:}R_{COA} \bullet \cot\left(DCA_{COA}\right) \bullet \Delta CLK_SF$$

For given image/scene point pair, the composite RGAZ error is computed as the sum of the contributions of the components. The sensitivity matrices are computed for the COA projection geometry. Scale factors cRGSF and cAZSF are defined for a more concise expression.

$$\Delta\mathbf{RGAZ} = \begin{bmatrix} \Delta RG \\ \Delta AZ \end{bmatrix} = \underbrace{\mathbf{M}_{\Delta ARP}^{RGAZ} \bullet \Delta\mathbf{ARP}_{COA}^{SCP} + \mathbf{M}_{\Delta VARP}^{RGAZ} \bullet \Delta\mathbf{VARP}}_{\substack{\text{RGAZ Error Due To} \\ \text{ARP Position and Velocity Errors}}} +$$

$$\underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix} \bullet \Delta R_BIAS + \begin{bmatrix} cRGSF \\ cAZSF \end{bmatrix} \bullet CLK_SF}_{\substack{\text{RGAZ Error Due To} \\ \text{Radar Sensor Errors}}} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix} \bullet \left(\Delta R_TROP + \Delta R_IONO \right)}_{\substack{\text{RG Error Due To} \\ \text{Propagation Delay Estimation Errors}}}$$

where: $cRGSF = -E{:}R_{COA}$ $\quad cAZSF = LOOK \bullet E{:}R_{COA} \bullet \cot\left(DCA_{COA}\right)$

12.4 Composite RGAZ Covariance

For a given image/scene projection pair, the expected error statistics of the composite RGAZ error may be used to compute the expected error statistics in the Image to Scene or Scene to Image projection. The Error Statistics parameter block allows two options for the set of parameters included with the image product. See Table 1-2. Option 1 provides an estimate of the composite RGAZ error covariance matrix. Option 2 provides an expanded set of estimated error statistics for the various components. An estimate of the composite RGAZ error covariance may be computed from the component statistics. For a product that includes both composite and component parameter sets (such a product is permitted but not expected), the component parameters should be used.

For Option 1, the estimated RGAZ covariance matrix is based on SCP projection geometry and is assumed to be constant for any image/scene point pair. While this is not precisely true, for most images and scene models, the RGAZ error varies only slightly across the image and the scene. The covariance for the given image/scene projection pair is set equal to the provided covariance.

Option 1: $\mathbf{C}_{RGAZ} = \mathbf{C}_{RGAZ}^{SCP}$

For Option 2, the estimated RGAZ covariance matrix is computed from the error statistics provided with the product and the projection geometry for the image/scene projection pair. The provided component error metadata parameters are used to compute the component error statistics listed in Table 1-2. Guidance is provided in the table for each parameter. For the optional parameters within the block that are omitted, a zero value is assumed.

Option 2: $\mathbf{C}_{RGAZ}$ - Computed as follows.

(1) The input position and velocity covariance matrix is transformed, as necessary, to be expressed in ECF coordinates. If the input frame is ECF, no transformation is required. If the input frame is RICF or RICI, form the needed 6 x 6 transformation matrix and then compute the ECF covariance. The RICF and RICI frames are computed at SCP COA time as defined in Section 3.2.

For FRAME = ECF: $$\mathbf{C}_{\text{ARP_PV}}^{\text{ECF}} = \mathbf{C}_{\text{ARP_PV}}^{\text{INPUT}}$$

For FRAME = RICF: $$\mathbf{C}_{\text{ARP_PV}}^{\text{ECF}} = \mathbf{T}_{\text{PV_RICF}}^{\text{PV_ECF}} \bullet \mathbf{C}_{\text{ARP_PV}}^{\text{INPUT}} \bullet \left(\mathbf{T}_{\text{PV_RICF}}^{\text{PV_ECF}}\right)^{\text{T}}$$

For FRAME = RICI: $$\mathbf{C}_{\text{ARP_PV}}^{\text{ECF}} = \mathbf{T}_{\text{PV_RICI}}^{\text{PV_ECF}} \bullet \mathbf{C}_{\text{ARP_PV}}^{\text{INPUT}} \bullet \left(\mathbf{T}_{\text{PV_RICI}}^{\text{PV_ECF}}\right)^{\text{T}}$$

Error rotation matrices: $$\mathbf{T}_{\text{PV_RICF}}^{\text{PV_ECF}} = \begin{bmatrix} \mathbf{T}_{\text{RICF}}^{\text{ECF}} & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_{\text{RICF}}^{\text{ECF}} \end{bmatrix} \qquad \mathbf{T}_{\text{PV_RICI}}^{\text{PV_ECF}} = \begin{bmatrix} \mathbf{T}_{\text{RICI}}^{\text{ECF}} & \mathbf{0} \\ \Omega_3 \bullet \mathbf{T}_{\text{RICI}}^{\text{ECF}} & \mathbf{T}_{\text{RICI}}^{\text{ECF}} \end{bmatrix}$$

Compute the RGAZ error covariance using the 2 x 6 sensitivity matrix for computing RGAZ error from the ARP position and velocity error in ECF for the image/scene point pair.

$$\mathbf{C}_{\text{RGAZ}}^{\text{ARP_PV}} = \mathbf{M}_{\text{ARP_PV}}^{\text{RGAZ}} \bullet \mathbf{C}_{\text{ARP_PV}}^{\text{ECF}} \bullet \left(\mathbf{M}_{\text{ARP_PV}}^{\text{RGAZ}}\right)^{\text{T}}$$

(2) The range bias error, ΔR_BIAS, contributes only a range error. Form the RGAZ error covariance from the input range bias variance, VAR_{RB}.

$$\mathbf{C}_{\text{RGAZ}}^{\text{RB}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \bullet \text{VAR}_{\text{RB}}$$

(3) Compute the RGAZ error covariance due to an error in the radar timing clock. The clock error is expressed as a scale factor error, CLK_SF. Form the matrix using the RGAZ error scale factors, cRGSF and cAZSF, and the input scale factor variance, $\text{VAR}_{\text{CLK_SF}}$.

$$\mathbf{C}_{\text{RGAZ}}^{\text{CLK_SF}} = \begin{bmatrix} \text{cRGSF}^2 & \text{cRGSF} \bullet \text{cAZSF} \\ \text{cRGSF} \bullet \text{cAZSF} & \text{cAZSF}^2 \end{bmatrix} \bullet \text{VAR}_{\text{CLK_SF}}$$

(4) The input variance for the error in troposphere propagation delay is scaled, as necessary, based upon the COA geometry for the image/scene projection pair. If both a SCP slant range delay variance, $\text{VAR}_{\text{RTS}}^{\text{SCP}}$, and a vertical delay variance, VAR_{RTV}, are provided, use the SCP slant range variance. Compute the expected variance for the range error due to the error in troposphere delay, ΔR_TROP.

$$\mathrm{VAR}_{\mathrm{TROP}} = \begin{cases} \mathrm{VAR}_{\mathrm{RTS}}^{\mathrm{SCP}}, & \text{if } \mathrm{VAR}_{\mathrm{RTS}}^{\mathrm{SCP}} \text{ is available} \\ \mathrm{VAR}_{\mathrm{RTV}} \bullet \dfrac{1}{\mathrm{sin_GRAZ}^2}, & \text{otherwise} \end{cases}$$

Note: For airborne collections, a variance estimate computed for the true collection geometry is recommended. Scaling a vertical path delay for grazing angles less than 10 degrees is not recommended.

(5) The input variance for the error in ionosphere propagation delay is scaled based upon the COA geometry for the image/scene projection pair. Compute the expected variance for the range error due to the error in troposphere delay, ΔR_IONO.

$$\mathrm{VAR}_{\mathrm{IONO}} = \mathrm{VAR}_{\mathrm{RIV}} \bullet \frac{1}{\mathrm{sin_GRAZ}^2}$$

Note: For airborne collections, this term should ALWAYS be zero.

(6) Form the RGAZ error covariance from the expected variances due to errors in estimated propagation delay, $\mathrm{VAR}_{\mathrm{TROP}}$ and $\mathrm{VAR}_{\mathrm{IONO}}$.

$$\mathbf{C}_{\mathrm{RGAZ}}^{\mathrm{PROP}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \bullet \left(\mathrm{VAR}_{\mathrm{TROP}} + \mathrm{VAR}_{\mathrm{IONO}} \right)$$

(7) Compute the composite RGAZ error covariance matrix as the sum of the component covariance matrices. All components are assumed to be statistically independent.

$$\mathbf{C}_{\mathrm{RGAZ}} = \underbrace{\mathbf{C}_{\mathrm{RGAZ}}^{\mathrm{ARP_PV}}}_{\substack{\text{RGAZ Error Stats} \\ \text{Due To ARP P\&V Error}}} + \underbrace{\mathbf{C}_{\mathrm{RGAZ}}^{\mathrm{RB}} + \mathbf{C}_{\mathrm{RGAZ}}^{\mathrm{CLK_SF}}}_{\substack{\text{RGAZ Error Stats} \\ \text{Due To Radar Sensor Errors}}} + \underbrace{\mathbf{C}_{\mathrm{RGAZ}}^{\mathrm{PROP}}}_{\substack{\text{RG Error Stats Due} \\ \text{To Prop Delay Errors}}}$$

12.5 Image To Scene Error Propagation

The Image to Scene projection error is defined to be the difference between the true scene position, **PT0**, minus the estimated scene point position, **E:PT0**. The true and estimated scene point positions are in ECF coordinates unless noted.

Image To Scene Projection Error: $$\underbrace{\Delta\mathbf{PT0}}_{\text{ERROR}} = \underbrace{\mathbf{PT0}}_{\substack{\text{TRUE}\\\text{VALUE}}} - \underbrace{\mathbf{E:PT0}}_{\substack{\text{ESTIMATED}\\\text{VALUE}}}$$

The projection error, **ΔPT0**, and its expected statistics are computed for the following sources of error.

ΔRGAZ Composite RGAZ image error for the given image/scene projection pair (components ΔRG and ΔAZ are in meters). The predicted error covariance is matrix $\mathbf{C}_{\text{RGAZ}}$. All covariance elements are in m^2.

ΔHAE Error in estimated scene surface Height Above The Ellipsoid (HAE) for a scene that is locally level in the neighborhood of the estimated scene point. Error is in meters and is the true height minus the estimated height: $\Delta\text{HAE} = \text{HAE0} - \text{E:HAE0}$. Predicted height error variance is VAR_{HAE}. Variance is in m^2.

ΔIL Error in estimated image location (components Δxrow and Δycol are in meters). Applicable when the initial image location **E:IL0** is estimated or measured for a distributed target or feature in the image. True image location is: **IL0** = **E:IL0** + **ΔIL**. The predicted error covariance is matrix $\mathbf{C}_{\text{IL}}$. All covariance elements are in m^2.

(1) The Image to Scene projection error due to the composite image error, **ΔRGAZ**, is computed as follows. See Figure 12-7. The error is a displacement in the ground plane. The error expressed in ground plane coordinates GPX and GPY is $\Delta\mathbf{GPXY}^{\text{RGAZ}}$. The error in ECF coordinates is $\Delta\mathbf{PT0}^{\text{RGAZ}}$.

$$\Delta\mathbf{GPXY}^{\text{RGAZ}} = \begin{bmatrix}\Delta\text{GPX0}\\ \Delta\text{GPY0}\end{bmatrix} = \mathbf{M}_{\text{SPXY}}^{\text{GPXY}} \bullet \mathbf{MIL}_{\text{RGAZ}}^{\text{SPXY}} \bullet \Delta\mathbf{RGAZ}$$

$$\Delta\mathbf{PT0}^{\text{RGAZ}} = \begin{bmatrix}\Delta\text{PT0_X}\\ \Delta\text{PT0_Y}\\ \Delta\text{PT0_Z}\end{bmatrix} = \mathbf{M}_{\text{GPXY}}^{\text{PT}} \bullet \Delta\mathbf{GPXY}^{\text{RGAZ}} = \mathbf{M}_{\text{GPXY}}^{\text{PT}} \bullet \mathbf{M}_{\text{SPXY}}^{\text{GPXY}} \bullet \mathbf{MIL}_{\text{RGAZ}}^{\text{SPXY}} \bullet \Delta\mathbf{RGAZ}$$

The expected error statistics for the error in ground plane coordinates and for the error in ECF coordinates, $\mathbf{C}_{\text{GPXY}}^{\text{RGAZ}}$ and $\mathbf{C}_{\text{PT}}^{\text{RGAZ}}$, are computed as follows. Covariance matrix $\mathbf{C}_{\text{GPXY}}^{\text{RGAZ}}$ is size 2 x 2 and, in general, will be of rank = 2. Covariance matrix $\mathbf{C}_{\text{PT}}^{\text{RGAZ}}$ is size 3 x 3 but will not be full rank.

$$\mathbf{C}_{\mathrm{GPXY}}^{\mathrm{RGAZ}} = \left(\mathbf{M}_{\mathrm{SPXY}}^{\mathrm{GPXY}} \bullet \mathbf{MIL}_{\mathrm{RGAZ}}^{\mathrm{SPXY}}\right) \bullet \mathbf{C}_{\mathrm{RGAZ}} \bullet \left(\mathbf{M}_{\mathrm{SPXY}}^{\mathrm{GPXY}} \bullet \mathbf{MIL}_{\mathrm{RGAZ}}^{\mathrm{SPXY}}\right)^{\mathrm{T}}$$

$$\mathbf{C}_{\mathrm{PT}}^{\mathrm{RGAZ}} = \mathbf{M}_{\mathrm{GPXY}}^{\mathrm{PT}} \bullet \mathbf{C}_{\mathrm{GPXY}}^{\mathrm{RGAZ}} \bullet \left(\mathbf{M}_{\mathrm{GPXY}}^{\mathrm{PT}}\right)^{\mathrm{T}}$$

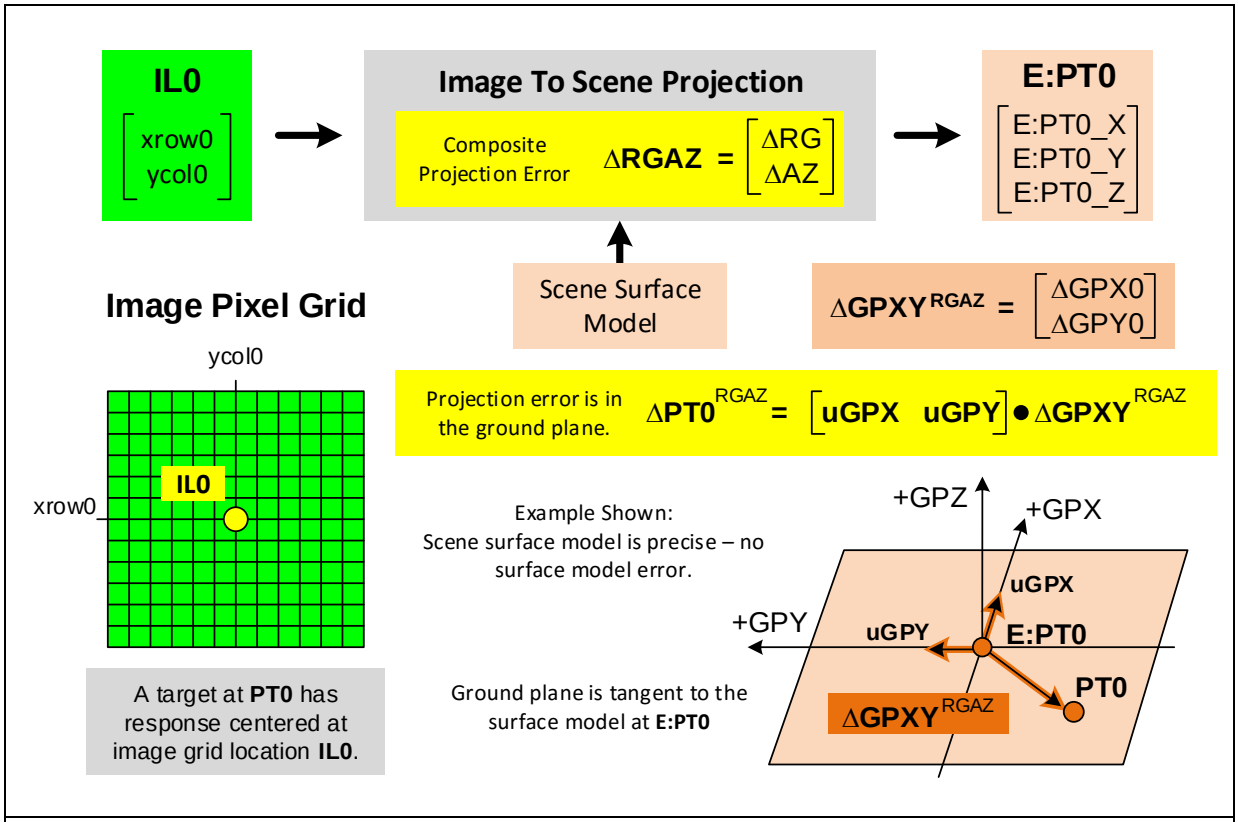


Figure 12-7 I2S Error Due To Composite RGAZ Error
The projection error is a displacement in the ground plane.

(2) The Image to Scene projection error due to an error in estimated scene height, ΔHAE, is computed as follows. Shown in Figure 12-8 is an example of the combined effects of image error and scene height error. The projection error due to the scene height error is $\Delta\mathbf{PT0}^{\mathrm{HAE}}$. The error is along the slant plane normal. Covariance matrix $\mathbf{C}_{\mathrm{PT}}^{\mathrm{HAE}}$ is size 3 x 3 but will be rank = 1. Note: The computed projection error assumes the scene is locally level near the true scene point.

$$\Delta\mathbf{PT0}^{\mathrm{HAE}} = \begin{bmatrix} \Delta\mathrm{PT0_X} \\ \Delta\mathrm{PT0_Y} \\ \Delta\mathrm{PT0_Z} \end{bmatrix} = \mathbf{M}_{\mathrm{HAE}}^{\mathrm{PT}} \bullet \Delta\mathrm{HAE}$$

$$\mathbf{C}_{\mathrm{PT}}^{\mathrm{HAE}} = \mathbf{M}_{\mathrm{HAE}}^{\mathrm{PT}} \bullet \left(\mathbf{M}_{\mathrm{HAE}}^{\mathrm{PT}}\right)^{\mathrm{T}} \bullet \mathrm{VAR}_{\mathrm{HAE}}$$

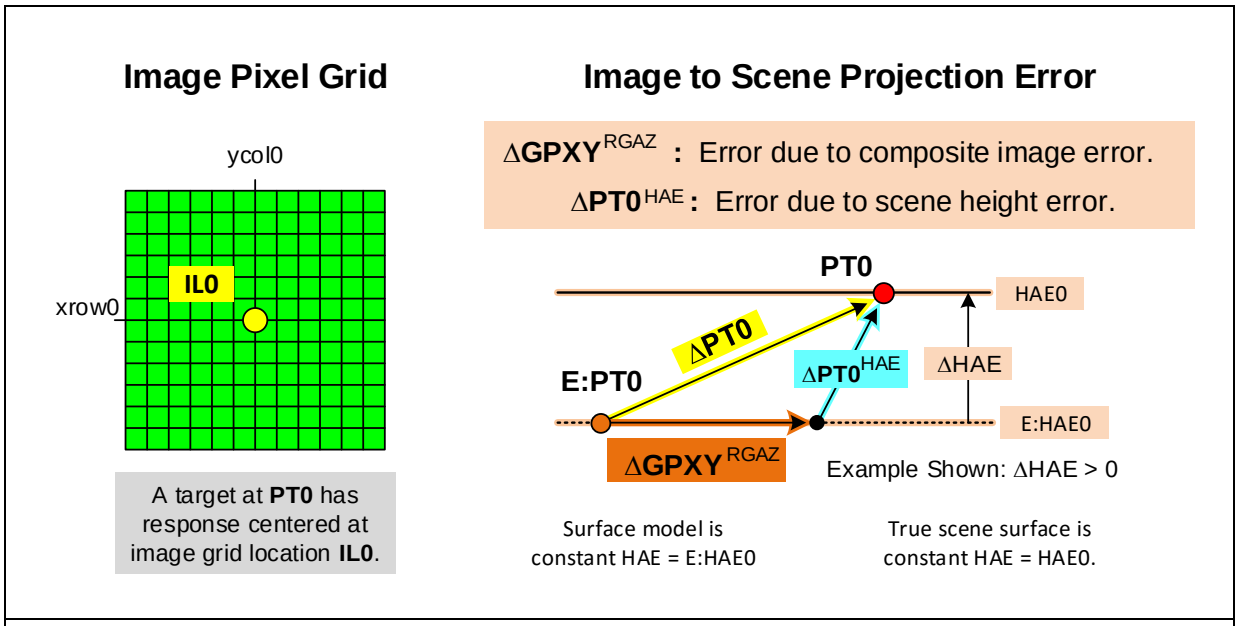


Figure 12-8 I2S Error Due To Image & Scene Height Errors
The error is the sum of the image error plus the effect of the surface height error.

(3) The Image to Scene projection error due to an error in the estimated image location, **ΔIL**, is computed as follows. Shown in Figure 12-9 is an example error in the estimated image grid location that results in an error in estimated scene point position. The error is a displacement in the ground plane. The error expressed in ground plane coordinates GPX and GPY is $\mathbf{\Delta GPXY}^{IL}$. The error in ECF coordinates is $\mathbf{\Delta PT0}^{IL}$.

$$\mathbf{\Delta GPXY}^{IL} = \begin{bmatrix} \Delta GPX0 \\ \Delta GPY0 \end{bmatrix} = \mathbf{M}_{IL}^{GPXY} \bullet \mathbf{\Delta IL}$$

$$\mathbf{\Delta PT0}^{IL} = \begin{bmatrix} \Delta PT0_X \\ \Delta PT0_Y \\ \Delta PT0_Z \end{bmatrix} = \mathbf{M}_{GPXY}^{PT} \bullet \mathbf{\Delta GPXY}^{IL} = \mathbf{M}_{GPXY}^{PT} \bullet \mathbf{M}_{IL}^{GPXY} \bullet \mathbf{\Delta IL}$$

The expected error statistics for the error in ground plane coordinates and for the error in ECF coordinates, $\mathbf{C}_{GPXY}^{IL}$ and $\mathbf{C}_{PT}^{IL}$, are computed as follows. Covariance matrix $\mathbf{C}_{GPXY}^{IL}$ is size 2 x 2 and, in general, will be of rank = 2. Covariance matrix $\mathbf{C}_{PT}^{IL}$ is size 3 x 3 but will not be full rank.

$$\mathbf{C}_{GPXY}^{IL} = \mathbf{M}_{IL}^{GPXY} \bullet \mathbf{C}_{IL} \bullet \left(\mathbf{M}_{IL}^{GPXY}\right)^{T}$$

$$\mathbf{C}_{PT}^{IL} = \mathbf{M}_{GPXY}^{PT} \bullet \mathbf{M}_{IL}^{GPXY} \bullet \mathbf{C}_{IL} \bullet \left(\mathbf{M}_{GPXY}^{PT} \bullet \mathbf{M}_{IL}^{GPXY}\right)^{T}$$

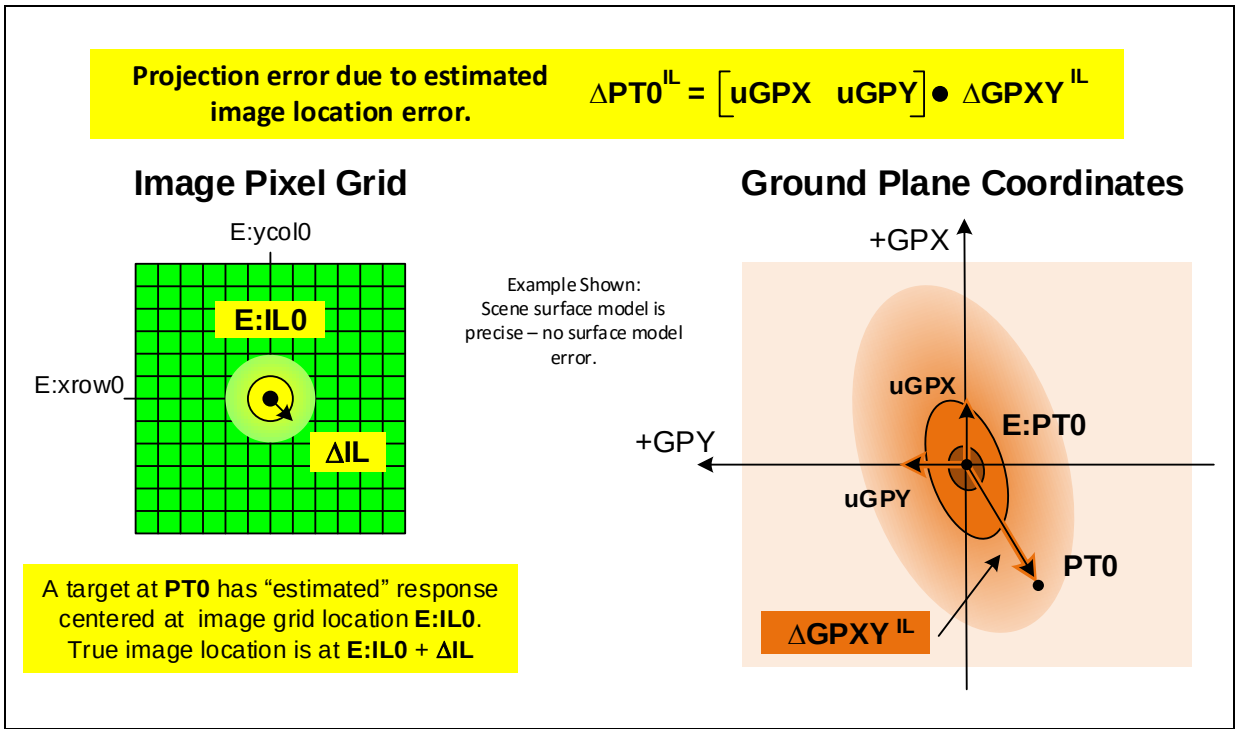


Figure 12-9 I2S Error Due To Image Location Error

An error in the estimated image location ΔIL leads to error in the computed scene point ΔPT0.

12.6 Scene to Image Error Propagation

The Scene to Image projection error is defined to be difference between the true image grid location, **IL0**, minus the estimated image grid location, **E:IL0**. The true and estimated grid locations are in image coordinates (components xrow0 and ycol0 are in meters).

Scene To Image Projection Error: $$\underbrace{\mathbf{\Delta IL0}}_{\text{ERROR}} = \underbrace{\mathbf{IL0}}_{\text{TRUE VALUE}} - \underbrace{\mathbf{E:IL0}}_{\text{ESTIMATED VALUE}}$$

The projection error, **ΔIL0**, and its expected statistics are computed for the following sources of error.

ΔRGAZ Composite RGAZ image error for the given image/scene projection pair (components ΔRG and ΔAZ are in meters). The predicted error covariance is matrix $\mathbf{C}_{RGAZ}$. All covariance elements are in m^2.

ΔPT Error in estimated scene point position in ECF coordinates**.** Applicable when the initial scene point position is an estimated position, **E:PT0**. True scene position is: **PT0** = **E:PT0** + **ΔPT**. The predicted error covariance is matrix $\mathbf{C}_{PT}$. All covariance elements are in m^2.

(1) The Scene To Image projection error due to the composite image error, **ΔRGAZ**, is computed as follows. The error in estimated image location is in image coordinates xrow and ycol is $\mathbf{\Delta IL0}^{\mathrm{RGAZ}}$.

$$\Delta\mathbf{IL0}^{\mathrm{RGAZ}} = \begin{bmatrix} \Delta\mathrm{xrow0} \\ \Delta\mathrm{ycol0} \end{bmatrix} = \mathbf{M}_{\mathrm{SPXY}}^{\mathrm{IL}} \bullet \mathbf{MPT}_{\mathrm{RGAZ}}^{\mathrm{SPXY}} \bullet \Delta\mathbf{RGAZ}$$

The expected error statistics for the error in image coordinates, $\mathbf{C}_{\mathrm{IL}}^{\mathrm{RGAZ}}$, is computed as follows. Covariance matrix $\mathbf{C}_{\mathrm{IL}}^{\mathrm{RGAZ}}$ is size 2 x 2 and, in general, will be of rank = 2.

$$\mathbf{C}_{\mathrm{IL}}^{\mathrm{RGAZ}} = \left(\mathbf{M}_{\mathrm{SPXY}}^{\mathrm{IL}} \bullet \mathbf{MPT}_{\mathrm{RGAZ}}^{\mathrm{SPXY}}\right) \bullet \mathbf{C}_{\mathrm{RGAZ}} \bullet \left(\mathbf{M}_{\mathrm{SPXY}}^{\mathrm{IL}} \bullet \mathbf{MPT}_{\mathrm{RGAZ}}^{\mathrm{SPXY}}\right)^{\mathrm{T}}$$

(2) The Scene To Image projection error due to an error in the estimated scene point position, **ΔPT**, is computed as follows. See Figure 12-10. The error in estimated image location is $\Delta\mathbf{IL0}^{\mathrm{PT}}$.

$$\Delta\mathbf{IL0}^{\mathrm{PT}} = \begin{bmatrix} \Delta\mathrm{xrow0} \\ \Delta\mathrm{ycol0} \end{bmatrix} = \mathbf{M}_{\mathrm{SPXY}}^{\mathrm{IL}} \bullet \mathbf{M}_{\mathrm{PT}}^{\mathrm{SPXY}} \bullet \mathbf{\Delta PT}$$

The expected error statistics for the error in image coordinates, $\mathbf{C}_{\mathrm{IL}}^{\mathrm{PT}}$, is computed as follows. Covariance matrix $\mathbf{C}_{\mathrm{IL}}^{\mathrm{PT}}$ is size 2 x 2 and, in general, will be of rank = 2.

$$\mathbf{C}_{\mathrm{IL}}^{\mathrm{PT}} = \left(\mathbf{M}_{\mathrm{SPXY}}^{\mathrm{IL}} \bullet \mathbf{M}_{\mathrm{PT}}^{\mathrm{SPXY}}\right) \bullet \mathbf{C}_{\mathrm{PT}} \bullet \left(\mathbf{M}_{\mathrm{SPXY}}^{\mathrm{IL}} \bullet \mathbf{M}_{\mathrm{PT}}^{\mathrm{SPXY}}\right)^{\mathrm{T}}$$

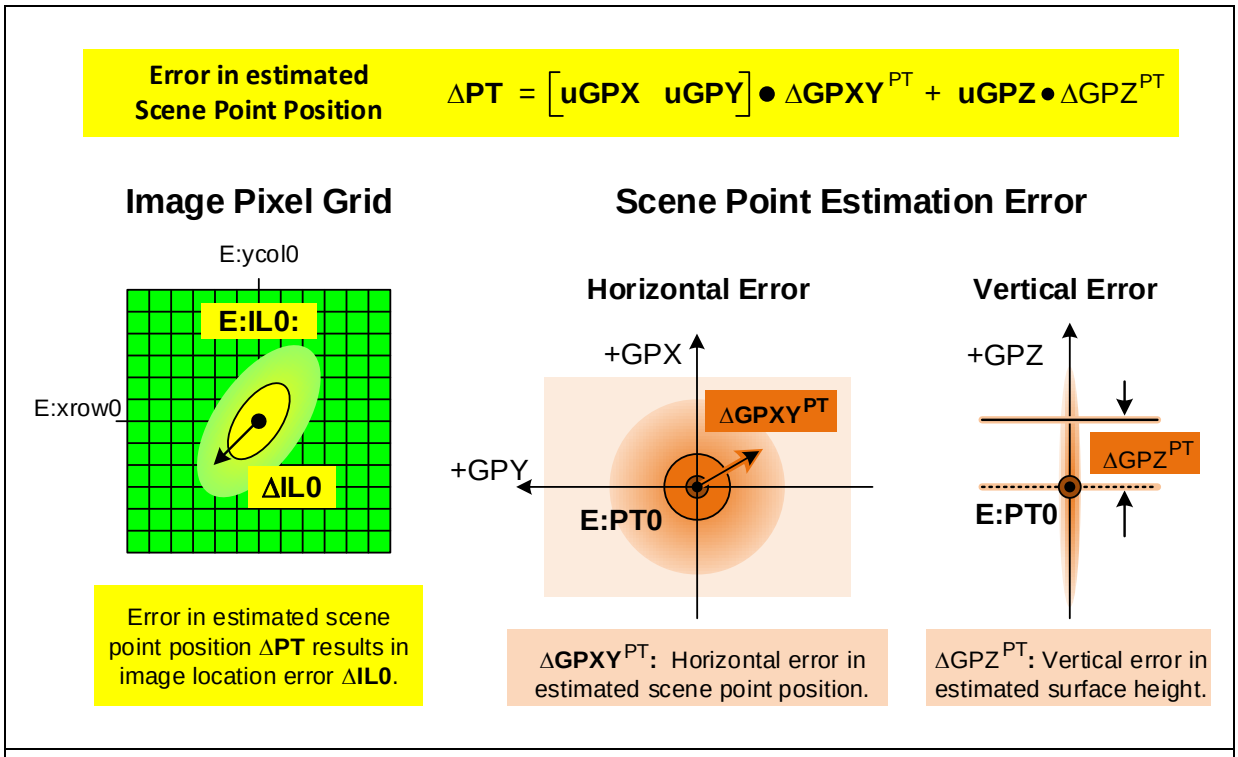


Figure 12-10 S2I Error Due To Scene Point Position Error

An error in the estimated position ΔPT0 leads to error in the computed image location ΔIL0.

12.7 Error Propagation Summary

For an ideal scatter at true scene point position **PT0**, the image signal response is centered at image grid location **IL0**. The method for computing projection errors and the expected error statistics are summarized below. For all the parameters, the error is defined as the true value minus the estimated value.

Error Convention For All Parameters: $$\underbrace{\Delta X}_{\text{ERROR}} = \underbrace{X}_{\substack{\text{TRUE}\\\text{VALUE}}} - \underbrace{E{:}X}_{\substack{\text{ESTIMATED}\\\text{VALUE}}}$$

Image To Scene Projection Error

The method for computing the error in the Image To Scene projection is summarized below. See Section 12.5. The initial image grid location may be the true image location, **IL0**, or an estimate of the location, **E:IL0**, that may include an error, Δ**IL**.

$$\mathbf{IL0} = \mathbf{E{:}IL0} + \Delta\mathbf{IL}$$

The image to scene projection of the image location, either **IL0** or **E:IL0**, yields estimated scene point **E:PT0**. The error in estimated scene point is **ΔPT0**.

Image To Scene Projection Error: $$\underbrace{\Delta\mathbf{PT0}}_{\text{ERROR}} = \underbrace{\mathbf{PT0}}_{\substack{\text{TRUE}\\\text{VALUE}}} - \underbrace{\mathbf{E:PT0}}_{\substack{\text{ESTIMATED}\\\text{VALUE}}}$$

Image To Scene Error Sources:

ΔRGAZ Composite RGAZ error for the image/scene projection pair (m). The predicted error covariance is matrix $\mathbf{C}_{RGAZ}$ (m^2). See Section 12.4.

ΔHAE Error in estimated scene Height Above The Ellipsoid (HAE) (m). Predicted height error variance is VAR_{HAE} (m^2).

Δ**IL** Error in estimated image grid location when the initial image location is an estimated value **E:IL0**. The predicted error covariance is matrix $\mathbf{C}_{IL}$ (m^2).

Image To Scene Error, ΔPT0 (ECF):

$$\Delta\mathbf{PT0} = \begin{bmatrix} \Delta PT0_X \\ \Delta PT0_Y \\ \Delta PT0_Z \end{bmatrix} = \Delta\mathbf{PT0}^{RGAZ} + \Delta\mathbf{PT0}^{HAE} + \Delta\mathbf{PT0}^{IL}$$

$$\Delta\mathbf{PT0} = \underbrace{\mathbf{M}_{\mathrm{GPXY}}^{\mathrm{PT}} \bullet \mathbf{M}_{\mathrm{SPXY}}^{\mathrm{GPXY}} \bullet \mathbf{MIL}_{\mathrm{RGAZ}}^{\mathrm{SPXY}} \bullet \Delta\mathbf{RGAZ}}_{\substack{\text{Scene Pt Error Due} \\ \text{To Composite RGAZ Image Error}}} + \underbrace{\mathbf{M}_{\mathrm{HAE}}^{\mathrm{PT}} \bullet \Delta\mathrm{HAE}}_{\substack{\text{Scene Pt Error Due} \\ \text{To Surface Height Error}}} + \underbrace{\mathbf{M}_{\mathrm{GPXY}}^{\mathrm{PT}} \bullet \mathbf{M}_{\mathrm{IL}}^{\mathrm{GPXY}} \bullet \Delta\mathbf{IL}}_{\substack{\text{Scene Pt Error Due} \\ \text{To Estimated Image Location Error}}}$$

Image To Scene Error Covariance, $\mathbf{C}_{\mathrm{PT}}$:

$$\mathbf{C}_{\mathrm{PT}} = \underbrace{\mathbf{C}_{\mathrm{PT}}^{\mathrm{RGAZ}}}_{\substack{\text{Statistics Due To} \\ \text{Composite Image Error}}} + \underbrace{\mathbf{C}_{\mathrm{PT}}^{\mathrm{HAE}}}_{\substack{\text{Statistics Due To} \\ \text{Surface Height Error}}} + \underbrace{\mathbf{C}_{\mathrm{PT}}^{\mathrm{IL}}}_{\substack{\text{Statistics Due To} \\ \text{Image Location Error}}}$$

$$\mathbf{C}_{\mathrm{PT}}^{\mathrm{RGAZ}} = \left(\mathbf{M}_{\mathrm{GPXY}}^{\mathrm{PT}} \bullet \mathbf{M}_{\mathrm{SPXY}}^{\mathrm{GPXY}} \bullet \mathbf{MIL}_{\mathrm{RGAZ}}^{\mathrm{SPXY}}\right) \bullet \mathbf{C}_{\mathrm{RGAZ}} \bullet \left(\mathbf{M}_{\mathrm{GPXY}}^{\mathrm{PT}} \bullet \mathbf{M}_{\mathrm{SPXY}}^{\mathrm{GPXY}} \bullet \mathbf{MIL}_{\mathrm{RGAZ}}^{\mathrm{SPXY}}\right)^{\mathrm{T}}$$

$$\mathbf{C}_{\mathrm{PT}}^{\mathrm{HAE}} = \mathbf{M}_{\mathrm{HAE}}^{\mathrm{PT}} \bullet \left(\mathbf{M}_{\mathrm{HAE}}^{\mathrm{PT}}\right)^{\mathrm{T}} \bullet \mathrm{VAR}_{\mathrm{HAE}}$$

$$\mathbf{C}_{\mathrm{PT}}^{\mathrm{IL}} = \mathbf{M}_{\mathrm{GPXY}}^{\mathrm{PT}} \bullet \mathbf{M}_{\mathrm{IL}}^{\mathrm{GPXY}} \bullet \mathbf{C}_{\mathrm{IL}} \bullet \left(\mathbf{M}_{\mathrm{GPXY}}^{\mathrm{PT}} \bullet \mathbf{M}_{\mathrm{IL}}^{\mathrm{GPXY}}\right)^{\mathrm{T}}$$

Scene To Image Projection Error

The method for computing the error in the Scene To Image projection is summarized below. See Section 12.6. The initial scene point position may be the true position, **PT0**, or an estimate of the position, **E:PT0**, that may include an error, Δ**PT**.

$$\mathbf{PT0} = \mathbf{E{:}PT0} + \Delta\mathbf{PT}$$

The scene to image projection of the scene point position, either **PT0** or **E:PT0**, yields estimated image location **E:IL0.** The error in estimated image location is **ΔIL0**.

Scene To Image Projection Error: $\underbrace{\mathbf{\Delta IL0}}_{\text{ERROR}} = \underbrace{\mathbf{IL0}}_{\substack{\text{TRUE} \\ \text{VALUE}}} - \underbrace{\mathbf{E:IL0}}_{\substack{\text{ESTIMATED} \\ \text{VALUE}}}$

Scene To Image Error Sources:

ΔRGAZ Composite RGAZ error for the image/scene projection pair (m). The predicted error covariance is matrix $\mathbf{C}_{\mathrm{RGAZ}}$ (m^2). See Section 12.4.

Δ**PT** Error in estimated scene point position when the initial position is an estimated value **E:PT0.** True position is: $\mathbf{E{:}PT0} + \Delta\mathbf{PT}$. Error Δ**PT** is in ECF coordinates (meters). The predicted error covariance is matrix $\mathbf{C}_{\mathrm{PT}}$ (m^2).

Scene To Image Error, ΔIL0:

$$\Delta\mathbf{IL0} = \begin{bmatrix} \Delta\text{xrow0} \\ \Delta\text{ycol0} \end{bmatrix} = \Delta\mathbf{IL0}^{\text{RGAZ}} + \Delta\mathbf{IL0}^{\text{PT}}$$

$$\Delta\mathbf{IL0} = \underbrace{\mathbf{M}_{\text{SPXY}}^{\text{IL}} \bullet \mathbf{MPT}_{\text{RGAZ}}^{\text{SPXY}} \bullet \Delta\mathbf{RGAZ}}_{\substack{\text{Image Location Error Due} \\ \text{To Composite RGAZ Image Error}}} + \underbrace{\mathbf{M}_{\text{SPXY}}^{\text{IL}} \bullet \mathbf{M}_{\text{PT}}^{\text{SPXY}} \bullet \Delta\mathbf{PT}}_{\substack{\text{Image Location Error Due} \\ \text{To Estimated Scene Pt Error}}}$$

Scene To Image Error Covariance, $\mathbf{C}_{\text{IL}}$:

$$\mathbf{C}_{\text{IL}} = \underbrace{\mathbf{C}_{\text{IL}}^{\text{RGAZ}}}_{\substack{\text{Statistics Due To} \\ \text{Composite Image Error}}} + \underbrace{\mathbf{C}_{\text{IL}}^{\text{PT}}}_{\substack{\text{Statistics Due To} \\ \text{Scene Pt Estimate Error}}}$$

$$\mathbf{C}_{\text{IL}}^{\text{RGAZ}} = \left(\mathbf{M}_{\text{SPXY}}^{\text{IL}} \bullet \mathbf{MPT}_{\text{RGAZ}}^{\text{SPXY}}\right) \bullet \mathbf{C}_{\text{RGAZ}} \bullet \left(\mathbf{M}_{\text{SPXY}}^{\text{IL}} \bullet \mathbf{MPT}_{\text{RGAZ}}^{\text{SPXY}}\right)^{\text{T}}$$

$$\mathbf{C}_{\text{IL}}^{\text{PT}} = \left(\mathbf{M}_{\text{SPXY}}^{\text{IL}} \bullet \mathbf{M}_{\text{PT}}^{\text{SPXY}}\right) \bullet \mathbf{C}_{\text{PT}} \bullet \left(\mathbf{M}_{\text{SPXY}}^{\text{IL}} \bullet \mathbf{M}_{\text{PT}}^{\text{SPXY}}\right)^{\text{T}}$$