NGA.SIG.0025_1.0_EGM
2019-08-20

NGA STANDARDIZATION DOCUMENT

Earth Gravitational Model and Geoid Heights
(2019-08-20)

Version 1.0

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NATIONAL CENTER FOR GEOSPATIAL INTELLIGENCE STANDARDS

Forward

This Standardization Implementation Guidance (SIG) document is approved for use by all Departments and Agencies of the Department of Defense.

Comments, suggestions, or questions on this document should be addressed to the

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Change Log

Version	Approval Date	POC	Change Description
1.0	2019-03-01		Initial release

TBR/TBD

TBD/TBR	Paragraph	Description

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1. Introduction

1.1. Scope

This Standardization Implementation Guidance (SIG) document is designed to provide guidelines for the preparation and use of the Earth Gravitational Model (EGM) and geoid heights derived from the model including the accuracy and limitations, and provide implementation guidance for the model. This document also provides guidelines for the preparation and use of computer programs and subroutines that compute the gravimetric components of the model. The EGM is one of the models incorporated in the World Geodetic System 1984 (WGS 84).

1.2. Applicability

The determination and application of geoid heights from the EGM as described in this SIG to model orthometric heights and global mean sea level may be used for all National System for Geospatial-Intelligence (NSG) geospatial-intelligence (GEOINT) requirements.

For the application of the EGM for satellite orbit determination, computation of other gravimetric components derived from the EGM such as deflection of the vertical to be applied to all other NSG systems, contact the National Geospatial-Intelligence Agency as described in the Forward.

1.3. Classification

This SIG and all information contained within are Unclassified.

2. References

The documents listed below are not necessarily all of the documents referenced herein, but are those needed to understand the information provided by this SIG document.

2.1. DoD specifications, standards, and handbooks

- NATO Standard GeoP-21, Geodetic Datums, Projections, Grids and Grid References, Edition A, Version 1, February 2016

- MIL-STD-2401, Department of Defense World Geodetic System, 11 January 1994

- CJCSI 3900.01D, Position (Point And Area) Reference Procedures, 14 May 2015

- NGA.STND.0036_1.0.0_WGS84, Department of Defense World Geodetic System, 2014-07-08

- NGA.STND.0019_2.0: Time-Space-Position Information (TSPI), 2012-04-05, Version 2.0

2.2. Non-DoD and non-government publications

- Pavlis, N. K., S. A. Holmes, S. C. Kenyon, *and* J. K. Factor (2012), The development and evaluation of the Earth Gravitational Model 2008 (EGM2008), J. Geophys. Res., 117, *B04406, doi:*10.1029/2011JB008916.

- F. G. Lemoine, et.al., The Development of the Joint NASA GSFC and NIMA Geopotential Model EGM96, NASA Goddard Space Flight Center, July 1998.

2.3. Order of precedence

Unless otherwise noted herein, in the event of a conflict between the text of this document and the references cited herein, the text of this document takes precedence. Nothing in this document, however, supersedes applicable laws and regulations unless a specific exemption has been obtained.

3. Terms, Acronyms, and Notation

3.1. Terms

The terms used in this document are given in Table 1.

Table 1 – Terms applicable to this document

earth gravitational model	A mathematical model describing the potential of gravity of the earth and used to compute various gravimetric related quantities.
ellipsoid height (geodetic height)	The distance of a point from the ellipsoid measured along the perpendicular from the ellipsoid to this point, positive if upwards or outside of the ellipsoid. [NGA.STND.0019_2.0 and ISO 19111:2007]
geoid	An equipotential surface of the Earth's gravity field which is everywhere perpendicular to the direction of gravity and which best fits mean sea level either locally or globally. [NGA.STND.0019_2.0 and ISO 19111:2007] A geoid is derived from an earth gravitational model.
geoid height (geoid undulation)	The height of the geoid above or below the referenced ellipsoid.
height	The distance of a point from a chosen reference surface measured upward along a line perpendicular to that surface. [NGA.STND.0019_2.0 and ISO 19111:2007]
mean sea level	The average level of the surface of the sea over all stages of tide and seasonal variations. [NGA.STND.0019_2.0 and ISO 19111:2007]
orthometric height	The distance of a point from the geoid surface measured along the perpendicular from the geoid to this point, positive if upwards or outside of the geoid. Used as the best approximation to height above (or below) mean sea level.

3.2. Acronyms

The acronyms used in this document are given in Table 2.

Table 2 – Acronyms applicable to this document

ASCII	American Standard Code for Information Interchange
DoD	Department of Defense
EGM	earth gravitational model
GEOINT	geospatial intelligence
MSL	mean sea level
NGA	National Geospatial-Intelligence Agency
NSG	National System for Geospatial-Intelligence
WGS 84	World Geodetic System 1984

3.3. Geodetic notation

The geodetic notation used in this document are given in Table 3.

Table 3 – Geodetic notation applicable to this document

Symbol	Quantity	Units
h	ellipsoid height (or geodetic height)	meters
H	orthometric height	meters
N	geoid height (or geoid undulation)	meters
V	gravitational potential function	meters^2/second^2

4. Description

4.1. General

The Earth is surrounded by its own gravitational field, which exerts an attractive force on all objects. As a first approximation, the strength of this field at any given point is proportional to the Earth's mass and inversely proportional to the square of the distance from the center of the Earth.

$$F \propto \frac{M_E}{r^2}$$

An EGM represents the external gravitational potential of the Earth on a global, regional, or local scale, and is usually expressed in the form of a set of spherical harmonic coefficients. The EGM is built from various types of observed and modeled terrestrial, airborne, and satellite gravity data. Development and use of regional and local scale EGMs are specialized products and are not applicable to this specification.

4.2. Gravitational potential function

The gravitational potential function (V) is a spherical harmonic function with an associated set of geopotential coefficients. The gravitational potential function is defined in Appendix A. To satisfy NSG requirements the EGM is a spherical harmonic model of the Earth's gravitational potential realized in the WGS 84 Reference Frame and is a tide-free system. Not included in this model are annual, seasonal, daily and other temporal and geophysical fluctuations of the Earth's gravitational field, such as precipitation and hydrodynamic processes, tectonic and magma events, and lunar and solar gravitational effects. The References in Sec. 2.2 provide a through description of the development of EGMs. Numerous gravimetric quantities can be determined from the gravitational potential function. See Table 4 for a partial listing of gravimetric quantities that can be computed from the gravitational potential function.

4.3. Geoid height

The geoid height is defined as the height of the geoid above or below the referenced ellipsoid and it provides several uses within the DoD. One use is the transformation of ellipsoidal height from GPS data to orthometric heights (to represent mean sea level heights). Another use is the establishment of a geoid that forms the reference surface for a global vertical datum to reference GEOINT, especially in elevation data production and safety of navigation products. Table 5 lists the basic metadata of the geoid height. The geoid height is also known as geoid undulation and the term geoid height will be used for this specification.

The geoid height is the single most useful value derived from the EGM for use by DoD personnel, the navigation and weapon system community, the earth modeling and simulation community, and for mapping and charting production. The geoid height is also the most rigorously calculated value from the gravitational potential function and spherical harmonic coefficients, with the current WGS 84 EGM capturing nearly 99% of the geoid signal.

Table 4 – Partial Listing of Gravimetric Quantities

Quantity	Symbol	Units
Geoid height	N	m
Height Anomaly	ζ	m
Spherically approximated gravity anomaly	Δg	mGal
Spherically approximated north-south vertical deflection	ξ	arcsec
Spherically approximated east-west vertical deflection	η	arcsec
Radial gravity anomaly gradient	Δg_r	mGal/km
Radial height anomaly gradient	ζ_r	m/km
Gravity anomaly second radial derivative	Δg_{rr}	mGal/km2
Radial component of gravity disturbance	T_r	mGal
North-south component of gravity disturbance	T_y	mGal
East-west component of gravity disturbance	T_x	mGal
Second radial derivative of T	T_{rr}	E.U.
Second north-south derivative of T	T_{yy}	E.U.
Second east-west derivative of T	T_{xx}	E.U.
Linearly approximated gravity anomaly	Δg	mGal

Table 5 – Geoid height metadata

Gravimetric Quantity	Symbol	Units	Notes
geoid height	N	meters	also known as geoid undulation

4.4. Relationship between ellipsoid, orthometric, and geoid heights

In geodetic applications, three primary reference surfaces for the Earth are used: 1) the Earth's topographic surface, 2) an ellipsoid of revolution, which is a reference surface of purely mathematical nature, and 3) the geoid. See Fig. 1.

In common practice, an ellipsoid height is the distance a given point is above or below the ellipsoid surface, whereas a geoid height is the distance the geoid surface is above or below the ellipsoid surface. See Table 1 for precise definitions. Determination of a "mean sea level" is a difficult problem because of the many factors that affect sea level. Sea level varies considerably on several scales of time and distance and the extent of this variability is due to the fact that the sea is in constant motion, affected by the solar and lunar tides, wind, atmospheric pressure, local gravitational differences, temperature, and salinity. In geodetic applications, the geoid is then used to serve as the vertical reference surface to approximate mean sea level (MSL) heights and a height measured from the geoid to a point is called an orthometric height. In areas where elevation data are not available from conventional geodetic leveling, an approximation of MSL heights using orthometric heights from the geoid can be obtained from the equation listed in Table 6. This equation illustrates the determination of the orthometric height (H) of a point as a subtraction of the geoid height (N) from the ellipsoid height (h).

For the purpose of this specification and practical application for DoD usage, the ellipsoid (geodetic) height, the orthometric height, and the geoid height are assumed to be co-linear to each other and positive upward along the local Z-axis, negative downward (i.e. geodetic effects such as deflection of the vertical and plumb line curvature are not considered).

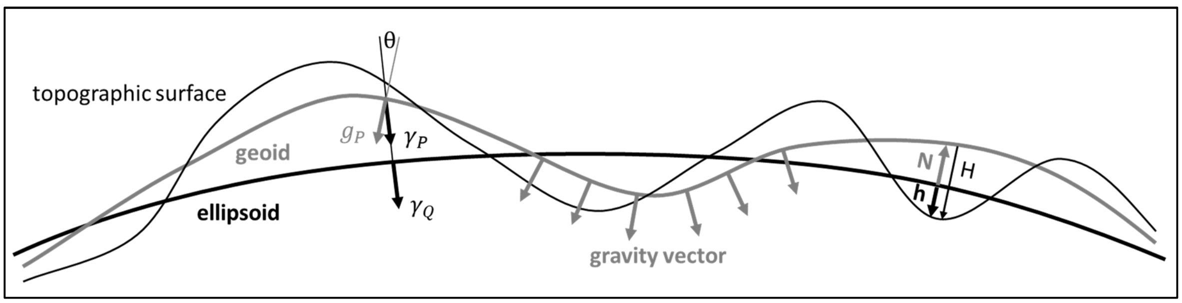

Fig. 1. Three primary reference surfaces for the earth

The topographic surface, the geoid surface and the ellipsoid surface. Indicated in the center are the gravity vector, perpendicular to the geoid surface. Also depicted are the normal and anomalous gravity vectors on the left and the various heights on the right.

Table 6 – Relationship between ellipsoid, orthometric, and geoid heights

h = H + N	
where	
h	= ellipsoid (geodetic) height (height relative to the ellipsoid)
H	= orthometric height (height relative to the geoid)
N	= geoid height (undulation)

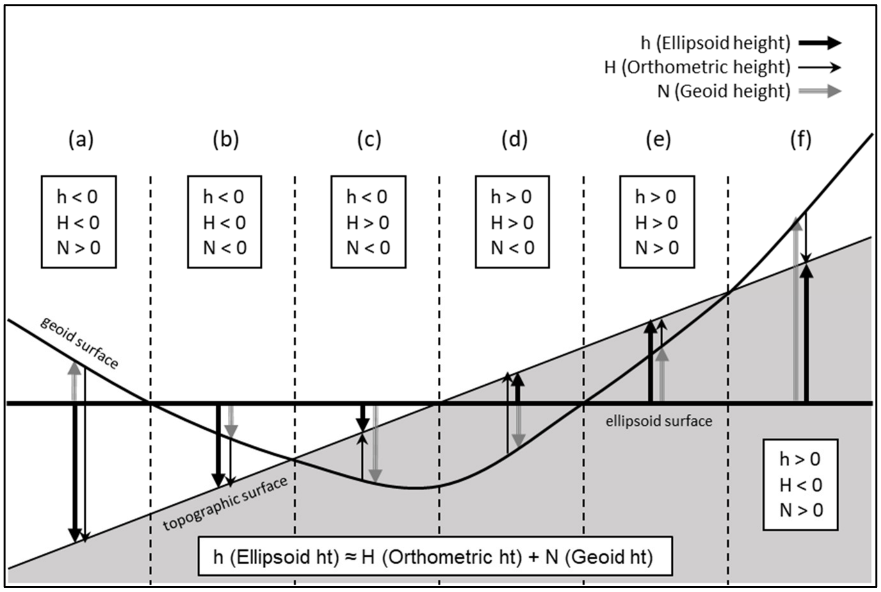

Fig. 2. The six combinations of the three geodetic surfaces

This figure depicts the six combinations of the three geodetic surfaces. When computing one value from the other two, care must be taken to ensure that the proper algebraic signs of the quantities are used in the equation. The arrows depicting the heights are separated for clarity, but in reality should be co-linear.

5. Specifications

5.1. Horizontal and vertical datums

The reference for latitudes and longitudes (horizontal datum) and the reference for heights (vertical datum) used by the EGM is the WGS 84. The reference ellipsoid for this datum is published in NGA.STND.0036_1.0.0_WGS84.

5.1.1. Orientation

The Earth's gravimetric field components are referenced to a local geodetic coordinate system, which is oriented such that the positive X-axis points East, the positive Y-axis points North, and the positive Z-axis points upward, as defined by the normal to the ellipsoid. Consequently, the negative X-axis points West, the negative Y-axis points South, and the negative Z-axis points downwards.

5.1.2. Horizontal position

The WGS 84 geodetic latitude ranges between –90 degrees at the South Pole to 90 degrees at the North Pole, while the WGS 84 geodetic longitude ranges from –180 degrees to 180 degrees, west and east respectively, from the WGS 84 Prime Meridian. Note: As described in Appendix A, by convention the gravitational potential function is defined using geocentric latitude. However, all EGM software and data file implementations for the NSG shall use WGS 84 geodetic latitude for input and output interfaces.

5.1.3. Horizontal units

The format for the geodetic coordinates for software and data file implementations are represented in degrees-decimal degrees format (+DD.dddddd for latitude, +DDD.dddddd for longitude). The degree portion will use leading zeros and leading sign.

5.1.4. Vertical direction

For the purpose of this specification and practical application for DoD usage, the ellipsoid (geodetic) height, the orthometric height, and the geoid height are assumed to be co-linear to each other and positive upward along the local Z-axis, negative downward (i.e. geodetic effects such as deflection of the vertical and plumb line curvature are not considered).

5.1.5. Vertical units

The format for the heights for software and data file implementations are in meters, reported to the nearest (rounded) millimeter (-MMM.mmm). The meter portion will not use leading zeros and will only be signed if negative.

5.2. Vertical aspect of model validity

Although the Earth's gravimetric field extends deep into the Earth and far out into space, the EGM, the data files and the associated software are valid only in the region from the surface of the geoid to 100 km above it. Above 100 km, truncated spherical harmonic coefficient files may be sufficient for orbit determinations, e.g. degree and order 70 for low earth orbit, degree and order 40 for medium earth orbit, and degree and order 18 for synchronous orbits.

5.3. Time datum

Each edition of the EGM shall have a specified base year (epoch). Unless otherwise specified or approved, each edition of the EGM will have a valid operational service life of ten (10) years forward from that time. Barring unforeseen circumstances, a new EGM will be released every 10 years, e.g. 2020, 2030, 2040 and the model epoch will be January 1 of that year.

5.4. Name

Each edition of the EGM shall be named based upon the release year, or epoch, and a suffix modifier may be included with the name to eliminate ambiguities, if required. As an example if an EGM was released in a particular year, say EGM2020 and an update was issued yet that year, the second EGM would be called EGM2020B.

5.5. EGM component accuracies

5.5.1. Definition of accuracy

When comparing an observed gravimetric component with the value predicted by the EGM, three errors come into play to account for the lack of agreement between the measured (observed) value, and the EGM supplied value: (i) measurement error, (ii) modeling error of commission, and (iii) modeling error of omission. These are explained in turn:

5.5.1.1. Measurement error

This category includes all errors due to the measurement itself, including errors due to instrument inaccuracy and temporal effects.

5.5.1.2. Modeling error of commission

This category deals with the error of the EGM in representing the geophysical phenomena it claims to model, e.g. the gravitational field model to degree and order 2159. The commission error directly depends on the availability of global terrestrial, airborne, and satellite gravity data and global elevation data.

5.5.1.3. Modeling error of omission

This category includes all natural contributions to the gravimetric field that are not included in the EGM. The omission error is because the EGM can only have a finite resolution, e.g. degree and order 2159, and will always omit a portion of the Earth's gravity field spectrum above this. Errors of omission are not represented in the EGM accuracy.

5.5.2. Geoid height (N) accuracy

The geoid height values generated by the EGM and its software implementations will have errors of commission smaller than ±0.20 meters in a root-mean-square (RMS) sense worldwide over the lifetime of the model.

5.6. Geoid height charts

As part of the delivery of an EGM, the producing agency will deliver a set of contour charts in digital format of the geoid heights (N) for the south polar, mid-latitudes, and north polar regions separately and are to include the capability to view the maps on-screen, print them as wall charts, or process them further by a Geographic Information System. The map projections recommended for the three regions are WGS 84 ellipsoid based South Polar Stereographic, Mercator, and North Polar Stereographic projections.

5.7. Technical report

The producing agency will provide a technical report describing the data sets used to build the model, the procedures employed to build the model from the data sets, and an estimate of the model's expected accuracy. The technical report will further provide estimates of EGM omission errors due to crustal and disturbance fields which are not represented in the EGM. The technical report will not be a part of this specification.

5.8. Security classification

The EGM spherical harmonic coefficients, grids, related computer software, charts, and documentation derived from the model are to be Unclassified, Public Release.

5.9. Media for product delivery

The mode of delivery of the EGM spherical harmonic coefficients, grids, computer software, and documents will be as digital downloads from NGA websites. An on-line EGM calculator may be provided by and available on NGA web sites.

6. Implementation Guidance

6.1. Intended use

The determination and application of geoid heights from the EGM as described in this SIG to model orthometric heights and global mean sea level may be used for all National System for Geospatial-Intelligence (NSG) geospatial-intelligence (GEOINT) requirements.

6.2. Universal implementation

Nearly every system and component of WGS 84 has nearly ubiquitous and universal implementation throughout the DoD, except for the Earth Gravitational Model and geoids. One purpose of the geoid is to perform a datum transformation between ellipsoid heights and orthometric heights. Numerous and incompatible implementations stem from many sources, including data file size and a general lack of understanding of the true nature of the geoid's transformation relationship between ellipsoid height and orthometric heights. Past implementation guidance for geoids recommended that the application determine their overall accuracy and then decimate the geoid grid to their accuracy, in essence, editing and changing the transformation. To drive to a common, universal implementation throughout the DoD, NGA recommends using the latest WGS 84 Earth Gravitational Model and the associated data files and software.

6.3. Updates to the EGM

It is important that systems implement the EGM information in a manner that will allow future updates of the data and software. The data and software will change with future refinements of the EGM, as improved gravimetric information becomes available to NGA, and this may necessitate updates to existing implementations as future operational accuracy requirements become known. Updates will nominally occur on a ten-year cycle starting in 2020. NGA recommends that DoD systems be designed to allow for updates to the EGM data and algorithms.

Table 7 – Anticipated EGM release dates

WGS 84 EGM	Release / Expected Release Date	Expected Legacy Date
EGM2008	Dec 2008	Dec 2019
EGM2020	Dec 2019	Dec 2029
EGM2030	Dec 2029	Dec 2030

6.4. Mean sea level and orthometric heights

For those systems that require an MSL height, NGA approves the use of orthometric heights derived from geoid heights as an approximation for MSL heights.

6.5. Spherical harmonics and interpolation grid methods

The EGM data and software, i.e. Spherical Harmonics and Interpolation Grid, described in this Implementation Guidance and available on NGA distribution services, produce geoid height results to within 0.01 meters of each other on a global scale. NGA approves of either method, spherical harmonics or interpolation grid, to compute geoid heights and is left to the discretion of system programmers. Some points to consider are:

- The spherical harmonic method produces a rigorous solution of the geoid height, where the interpolation grid method is an approximate solution of the geoid height.
- The interpolation grid method agrees with the spherical harmonic method to within 0.01 meters on a global scale.
- The spherical harmonic method requires two data files, where the interpolation grid requires one data file.
- The two spherical harmonic data files are only 10% larger in combined file size than the interpolation grid, ~375MB to ~335MB, respectively.
- The execution time for implementations of the spherical harmonic algorithm is usually slower than implementations of the interpolation grid method.
- Use of the grid interpolation method may provide better system integration due to the ubiquitous and universal understanding of interpolation schema and source code by system developers. Understanding of the internal source code logic by a system developer of the spherical harmonic method is more complex and may require advanced knowledge in physical geodesy and mathematics.

6.5.1. Geoid heights represented by spherical harmonics

The spherical harmonic coefficients and the associated software provide the most accurate and rigorous solution to the geoid height at a location.

6.5.2. Spherical harmonic coefficients

One or more spherical harmonic coefficient data files are required for the direct computations of geoid heights from the gravitational potential function. The coefficient files are defined in Appendix B.

- Note: NGA's present implementation for EGM96, EGM2008, and the anticipated implementation for EGM2020, each require two spherical harmonic coefficient data files, a Gravitational Coefficients file and a Gravitational Correction Coefficients file.

6.5.3. Spherical harmonic computer software

One or more software implementations of the spherical harmonic algorithm in one or more programming languages and system environments are to be part of the EGM. The EGM producer will provide a programmer's guide, an end user's guide, or an on-line help menu as appropriate, for each item of software delivered.

6.5.4. Geoid heights represented by interpolation grids

The interpolation grid and the associated software provide an approximated solution to the geoid height at a point. Depending on the grid spacing and software algorithm of the interpolation grid, geoid heights can be obtained that are within 0.01 meters to the rigorous spherical harmonic solution.

6.5.5. Interpolation grids

As an alternate method to spherical harmonics, a precompiled global grid of geoid values can provide geoid heights. The grid is derived by repeatedly using the spherical harmonic computations at predetermined grid points. The interpolation grid is defined in Appendix C.

6.5.6. Interpolation grid computer software

One or more software implementations of the grid interpolation algorithm in one or more programming languages and system environments are to be part of the EGM. The EGM producer will provide a programmer's guide, an end user's guide, or an on-line help menu as appropriate, for each item of software delivered. An on-line EGM calculator may be available on NGA web sites.

6.5.7. Summary of data file physical metadata

The following Table summarizes information about the various data files described in Appendices B and C. Note that when using the Spherical Harmonics method to compute geoid heights, both the Gravitational Coefficients file and the Gravitational Correction Coefficients file are required. When using the Interpolation Grid method, only the Interpolation Grid data file is required.

Table 8 – Summary of data file physical metadata

Spherical Harmonics Method	Data Format	Header Rows	Data Rows	Data Columns per Row	File Size
Gravitational Coefficients file	ASCII	~12	~2,401,333	~101	~236MB
Gravitational Correction Coefficients file	ASCII	~10	~2,336,041	~61	~139MB
Interpolation Grid Method	ASCII	2	~4321	~786,250	~335MB

6.6. Intended application of the EGM

6.6.1. Single point ellipsoid-orthometric height transformation

One intended application of the EGM is the transformation of single point ellipsoidal height from GPS data to orthometric heights, especially within GPS receivers for the purpose of sea, land, and air navigation.

Caution: Without further processing and analysis, aggregating numerous points calculated via this procedure may not provide accurate GEOINT data or present unknown errors. Some of these problems may be a lack of monotonicity of lakes and rivers, or shear effects with adjacent data sets.

6.6.2. Global vertical datum

Another application of the EGM is the establishment of a geoid that forms the reference surface for a global vertical datum to reference GEOINT, especially in elevation data production and safety of navigation products. Photogrammetric and cartographic techniques are required to accurately process and apply geoid height data to finish GEOINT products.

6.7. Single point EGM transformation versus point mass and product EGM transformation

The following section addresses the topic of using multiple geoids in an attempt to transform a single point, a small group of points, or an entire product from one EGM to another EGM.

Single Point transformation: Mathematical removal of a point geoid and the subsequent addition of a second point geoid usually does not present significant geodetic errors for a single point of GEOINT data. For example, the height of a water tower near an airfield is determined photogrammetrically with EGM96 used as the final vertical datum. A typical accuracy for this process could be several meters vertically. With the nominal global geoid height RMS accuracy for EGM96 and EGM2008 at 0.5 meter and 0.15 meter, respectively, the removal of EGM96 and then subsequent addition of EGM2008 does not significantly change the vertical accuracy of the water tower.

Multiple Point transformation: For example, an attempt to ‘transform’ EGM96 based Shuttle Radar Topography Mission data to EGM2008. SRTM data is produced using the EGM96 geoid for the vertical datum. Mathematically, the EGM96 geoid height for all the data points in an SRTM cell can be calculated and then mathematically subtracted from each data point, obtaining what appears to be a WGS 84 ellipsoid height. Then performing a reverse application of the geoid, the EGM2008 geoid height can be calculated for each data point again and mathematically added to the intermediate ellipsoid height, obtaining what would appear to be EGM2008 based SRTM data.

The fallacy of the previous method is that EGMs are not simply applied as a last step in the photogrammetric production of elevation data, but are an integral process of elevation production.

A. Appendix A - Gravitational Potential Function

A.1. Gravitational potential function (V)

The Earth's external gravitational potential at any point on or above the Earth's surface can be represented on a global scale by summing up a degree and order of coefficients of a spherical harmonic expansion. The earth's external gravitational potential, V, is given by:

Table A-1 – Gravitational potential function (V)

$$V = \frac{GM}{r}\left[1 + \sum_{n=2}^{n_{max}} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} \left(\overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda\right) \overline{P}_{nm}(\sin \phi')\right]$$

where

V	= gravitational potential
GM	= geocentric gravitational constant
r	= distance from the Earth's center of mass
a	= semi-major axis of the WGS 84 ellipsoid
n, m	= degree and order, respectively
ϕ'	= geocentric latitude
λ	= geocentric longitude = geodetic longitude
$\overline{C}_{nm}, \overline{S}_{nm}$	= fully normalized gravitational coefficients
$\overline{P}_{nm}(\sin \phi')$	= fully normalized associated Legendre function $= \left[\frac{(n-m)!\,(2n+1)k}{(n+m)!}\right]^{\frac{1}{2}} P_{nm}(\sin \phi')$
$P_{nm}(\sin \phi')$	= associated Legendre function $= (\cos \phi')^m \frac{d^m}{d(\sin \phi')^m} [P_n(\sin \phi')]$
$[P_n(\sin \phi')]$	= Legendre polynomial $= \frac{1}{2^n n!} \frac{d^n}{d(\sin \phi')^n} (\sin^2 \phi' - 1)^n$

Note:

$$\begin{vmatrix}\overline{C}_{nm} \\ \overline{S}_{nm}\end{vmatrix} = \left[\frac{(n+m)!}{(n-m)!\,(2n+1)k}\right]^{\frac{1}{2}} \begin{vmatrix}C_{nm} \\ S_{nm}\end{vmatrix}$$

where

C_{nm}, S_{nm}	= conventional (unnormalized) gravitational coefficients
for	$m = 0, k = 1$ $m \neq 0, k = 2$

Note: As described above, by convention the gravitational potential function is defined using geocentric latitude. However, all EGM software and data file implementations for the NSG shall use WGS 84 geodetic latitude for input and output interfaces.

B. Appendix B - Earth Gravitational Model Spherical Harmonic Data Files

B.1. Purpose

Numerous gravimetric quantities can be determined from the gravitational potential function and the EGM gravitational coefficients data file. For geoid height computations, where the full resolution of the EGM is sought, it is recommend to use the EGM Gravitational Coefficients to degree 2190 with the parallel use of the EGM Gravitational Correction Coefficients to degree 2160. An overview of the spherical harmonic calculations are listed in Chapter 6 of NGA.STND.0036_1.0.0_WGS84 with a brief explanation of the internal geodetic math performed and the need for an additional coefficient file to compute the geoid heights.

B.2. Gravitational coefficients file

The EGM Gravitational Coefficients data file consists of two parts, a Header section and a Data section. The Header section defines the parameters and structure of the gravitational model and the Data section provides the harmonic coefficients. The entire file is American Standard Code for Information Interchange (ASCII) and all elements are space delimited. Allowed characters are the ‘printable’ ASCII characters (decimal 32 – 126).

B.2.1. Gravitational coefficients header section

The Gravitational Coefficients header section has the following structure outlined in Table B-1. The only required parameters are the “begin_of_head” as the first parameter and “end_of_head” as the last parameter. The other parameter names can be listed in any order. A parameter name can exist with an associated blank or ‘null’ parameter value, however any parameter value must be paired with a parameter name. A blank or ‘null’ row is allowed.

B.2.2. Gravitational coefficients data section.

The Data section immediately follows the Header section has the following structure outlined in Table B-2. The Data section contains the unit-less, spherical harmonic coefficients of the Earth’s gravitational potential and the associated error standard deviations terms. The coefficients will represent an ellipsoidal harmonic model beginning at degree and order 2,0 and complete to degree and order 2159. The coefficients will then extend to degree 2190 and order 2159 as a spherical harmonic model. Missing and non-existent coefficients are written as zeroes. The Data section contains 2,401,333 ASCII formatted records, each record containing six space separated words: n{I5}, m{I5}, C_{nm}{E25.15}, S_{nm}{E25.15}, $C_{nm}sigma${E20.10}, $S_{nm}sigma${E20.10}.

B.2.3. Example Gravitational Coefficients File

Table B-3 is an example of the gravitational coefficients file.

Table B-1 – Description of gravitational coefficients header section

Parameter Name	Parameter value or description
begin_of_head===========	Required. The start of the header section, no data is on this line other than the Parameter name
model_name	The name of the model
product_type	The type of data in the file, example: gravity field
earth_gravity_constant	The earth gravitational constant used
radius	The semi-major axis distance
max_degree	The maximum degree of the model
errors	Information of the errors, examples: none calibrated formal
norm	Type of normalization used on the coefficients, examples: none semi-normalized fully normalized
tide_system	Tide system, examples: mean tide zero tide tide free
notes	Any notes to accompany the file, may be multiple lines
key	Describes the column structure of the data elements, examples: n: degree m: order C: conventional gravitational coefficient S: conventional gravitational coefficient sigma C: error term sigma S: error term
end_of_head===========	Required. The end of the header section, no data is on this line other than the Parameter name

Table B-2 – Description of gravitational coefficients data section

n{I5}	m{I5}	C_{nm}{E25.15}	S_{nm}{E25.15}	$C_{nm} sigma${E20.10}	$S_{nm} sigma${E20.10}
2	0	-0.123456789012345E-01	-0.123456789012345E-01	-0.1234567890E-01	-0.1234567890E-01
2190	2190	-0.123456789012345E-01	-0.123456789012345E-01	-0.1234567890E-01	-0.1234567890E-01

Table B-3 – Example gravitational coefficients file

```
begin_of_head==============================================================================
model_name                      EGM2020
product_type                    gravity field
earth_gravity_constant          3.986004418 × 10^+14  m^3 / s^2
radius                          6378137.0  m
max_degree                      2190
errors                          calibrated
norm                            fully normalized
tide_system                     tide free
notes                           This is an example of the file format for the EGM Gravitational Coefficients file
key                                 n           m            C              S            sigma C         sigma S
end_of_head===============================================================================
      2        0      -0.484165143790815E-03       0.000000000000000E+00     0.7481239490E-11     0.0000000000E+00
      2        1      -0.206615509074176E-09       0.138441389137979E-08     0.7063781502E-11     0.7348347201E-11
      2        2       0.243938357328313E-05      -0.140027370385934E-05     0.7230231722E-11     0.7425816951E-11
      3        0       0.957161207093473E-06       0.000000000000000E+00     0.5731430751E-11     0.0000000000E+00
      3        1       0.203046201047864E-05       0.248200415856872E-06     0.5726633183E-11     0.5976692146E-11
      3        2       0.904787894809528E-06      -0.619005475177618E-06     0.6374776928E-11     0.6401837794E-11
   ....     ....            ....                        ....                    ....                  ....
   2190     2187       0.000000000000000E+00       0.000000000000000E+00     0.0000000000E+00     0.0000000000E+00
   2190     2188       0.000000000000000E+00       0.000000000000000E+00     0.0000000000E+00     0.0000000000E+00
   2190     2189       0.000000000000000E+00       0.000000000000000E+00     0.0000000000E+00     0.0000000000E+00
   2190     2190       0.000000000000000E+00       0.000000000000000E+00     0.0000000000E+00     0.0000000000E+00
```

B.3. Gravitational correction coefficients file

The EGM Gravitational Corrections Coefficients data file consists of two parts, a Header section and a Data section. The Header section defines the parameters and structure of the gravitational model and the Data section provides the harmonic coefficients. The entire file is ASCII and all elements are space delimited. Allowed characters are the 'printable' ASCII characters (decimal 32 – 126).

B.3.1. Gravitational correction coefficients header section

The Gravitational Correction Coefficients file header section has the following structure outlined in Table B.4. The only required parameters are the "begin_of_head" as the first parameter and "end_of_head" as the last parameter. The other parameter names can be listed in any order. A parameter name can exist with an associated blank or 'null' parameter value, however any parameter value must be paired with a parameter name. A blank or 'null' row is allowed.

Table B-4 – Description of gravitational correction coefficients header section

Parameter Name	Parameter value or description
`begin_of_head===========`	The start of the header section, no data is on this line other than the Parameter name
`model_name`	The name of the model
`product_type`	The type of data in the file, example: correction coefficients
`earth_gravity_constant`	The earth gravitational constant used,
`radius`	The semi-major axis distance
`max_degree`	The maximum degree of the model
`tide_system`	Tide system: mean tide, zero tide, tide free
`notes`	Any notes to accompany the file, may be multiple lines
`key`	The structure of the data elements, examples: n: degree m: order CC: correction coefficient CS: correction coefficient
`end_of_head===========`	The end of the header section, no data is on this line other than the Parameter name

B.3.2. Gravitational correction coefficients data section

The data section immediately follows the header section has the following structure outlined in Table B-5. The data section contains spherical harmonic coefficients of the Height Anomaly-to-Geoid Undulation correction (conversion) term $\{CC_{nm}, CS_{nm}\}$ in units of meters. These corrections are applied to EGM height anomalies computed on the WGS 84 ellipsoid, to yield EGM geoid heights with respect to WGS 84. This model is complete to degree and order 2160. The file contains 2,336,041 ASCII formatted records, each record containing four space separated words: n{I5}, m{I5}, CC_{nm}{E25.15}, CS_{nm}{E25.15}.

Table B-5 – Description of gravitational correction coefficients data section

n{I5}	m{I5}	CC_{nm}{E25.15}	CS_{nm}{E25.15}
2	0	-0.123456789012345E-01	-0.123456789012345E-01
2160	2160	-0.123456789012345E-01	-0.123456789012345E-01

B.3.3. Example gravitational correction coefficients file

Table B-6 is an example of the gravitational correction coefficients file.

Table B-6 – Example gravitational correction coefficients file

```
begin_of_head==================================================
model_name                  EGM2020
product_type                correction coefficients
earth_gravity_constant      3.986004418 × 10^+14  m^3 / s^2
radius                      6378137.0  m
max_degree                  2160
tide_system                 tide free
notes                       This is an example of the file format for the
                            EGM Gravitational Corrections Coefficients
                            data file
key                             n           m           CC          CS
end_of_head====================================================
      0       0     -5.28571479946422E-02      0.00000000000000E+00
      1       0      2.11188858958660E-03      0.00000000000000E+00
      1       1     -9.94073787167880E-03     -2.26201527740839E-02
      2       0     -3.20244426540044E-02      0.00000000000000E+00
      2       1      1.84851439031423E-03     -2.30432815153383E-02
      2       2      2.78005479599099E-02     -3.46333405591105E-03
      3       0      4.86208611555534E-02      0.00000000000000E+00
   ....    ....          ....                       ....
   2160    2157     -4.11699876635155E-06      2.59896101222736E-06
   2160    2158     -2.69752312791186E-06     -4.01168867512804E-06
   2160    2159     -7.08040747120047E-06      4.11876014753279E-06
   2160    2160     -7.53437779189490E-07     -1.66275700756324E-06
```

C. Appendix C - Earth Gravitational Model Interpolation Grid Data File

C.1. Purpose

As an alternate to the Spherical Harmonic implementation, an interpolation grid data file can be constructed that contains precomputed point values of gravimetric values arranged in a regular grid with respect to WGS 84. The data is computed from the EGM spherical harmonics.

C.2. Interpolation grid

The Interpolation Grid data file consists of two parts, a Header section and a Data section. The Header section defines the parameters and structure of the grid file and the Data section provides the gridded data for interpolation. The entire file is ASCII and all elements are space delimited. Allowed characters are the 'printable' ASCII characters (decimal 32 – 126).

C.3. Interpolation grid header section

The Interpolation Grid data file has a 2 row header with the structure described below. All 'words' are required in the order listed, with a default value given for each, null or 'zero length' words are not allowed.

C.3.1. Row1 Word1: Model name

The name of the EGM used to create the Interpolation Grid data.

Examples:
- PGM2015
- EGM2020

Default: EGM

C.3.2. Row1 Word2: Release date

The release date of the data in the Interpolation Grid. This may not be the same date as the EGM release date, as the interpolation data may be computed days, months, or years after creating the EGM used.

Examples:
- 15DEC19
- 15_DEC_2019

Default: 01JAN01

C.3.3. Row1 Word3: Data type

The type of data represented in the Interpolation Grid. Numerous gravimetric quantities can be represented.

Examples:
- GEOID_HEIGHTS
- EAST-WEST_DOV
- GRAVITY_ANOMALY

Default: GEOID_HEIGHTS

C.3.4. Row1 Word4: Units

The units associated with the data type.

Examples:
- METERS
- ARCSECONDS
- MGALS

Default: METERS

C.3.5. Row1 Word5: Ellipsoid reference

The ellipsoid used to reference the data type.

Examples:

WGS_84
GRS80
MEAN_EARTH_ELLIPSOID

Default: WGS_84

C.3.6. Row1 Word6: Datum

The datum used to reference the data type.

Examples:

WGS_84
ITRF

Default: WGS_84

C.3.7. Row1 Word7: Tide system

The tide system associated with the EGM used.

Examples:

ZERO_TIDE
MEAN_TIDE
TIDE_FREE

Default: TIDE_FREE

C.3.8. Row1 Word8: Area of extent

The area of extent of the data type. Global implies -90 to 90 degrees of Latitude, inclusive, and 0 to 360 degree of Longitude, inclusive. Any subset of Global is considered Local.

Examples:

GLOBAL
LOCAL

Default: GLOBAL

C.3.9. Row1 Word9: Northern latitude limit

The northern latitude limit of the data type in geodetic degrees-decimal degrees to six decimals, including leading zeroes and leading sign. The Northern latitude limit must be greater than the Southern latitude limit. For a global model, this must be 90.000000.

Limits:

$-89.983333 \leq x \leq 90.00000$

Examples:

-05.500000
80.000000

Default: 90.000000

C.3.10. Row1 Word10: Southern latitude limit

The southern latitude limit of the data type in geodetic degrees-decimal degrees to six decimals, including leading zeroes and leading sign. The Southern latitude limit must be less than the Northern latitude limit. For a global model, this must be -90.000000.

Limits:

$-90.0000 \leq x \leq 89.983333$

Examples:

-60.000000
05.500000
Default: -90.000000

C.3.11. Row1 Word11: Western longitude limit

The western longitude limit of the data type in geodetic degrees-decimal degrees to six decimals, including leading zeroes and leading sign. The Western longitude limit must be less than the Eastern longitude limit. For a global model, this must be -180.000000.

Limits:
$-180.0000 \leq x \leq 179.983333$
Examples:
-015.500000
120.000000
Default: -180.000000

C.3.12. Row1Word12: Eastern longitude limit

The western longitude limit of the data type in geodetic degrees-decimal degrees to six decimals, including leading zeroes and leading sign. The Eastern longitude limit must be greater than the Western longitude limit. For a global model, this must be 180.000000.

Limits:
$-179.983333 \leq x \leq 180.0000$
Examples:
-015.500000
120.000000
Default: 180.000000

C.3.13. Row1 Word13: Grid spacing

The equiangular spacing in terms of geodetic coordinates of the data in minutes to three decimals, including leading zeroes.

Examples:
1.000
15.000
Default: 2.5

C.3.14. Row2 Word1: Notes

Row 2 of the Header section may contain free text that provides further information about the data file to users. This information is not used directly by the software for the calculation of the geoid heights.

Default: Notes

C.4. Interpolation grid data section

The Data section immediately follows the Header section, Row 3, and contains the grid data as described in the Header. This section holds the 2.5 minute x 2.5 minute global grid (equiangular spacing in terms of WGS 84 geodetic coordinates) geoid heights in a space delimited ASCII format. Each record contains all of the geoid heights for a single parallel band. The first record in the Data section contains the northern-most parallel, and the first value in each record is the western-most value for that parallel. Note that each geoid value situated on the 180 West (-180º) longitude, the first element of each record, is duplicated at the end of their respective record, at longitude of 180° East (180º), the last element of each record. As such, the Data Section contains 4321 rows x 8641 columns of geoid heights.

C.5. Example interpolation grid data file

Table C-1 is an example of the interpolation grid data file.

Table C-1 – Example interpolation grid data file

```
EGM2020 15DEC19 GEIOD_HEIGHTS METERS WGS_84 WGS_84 TIDE_FREE GLOBAL 90.0000 -90.0000 -180.0000 180.0000 2.500
Notes This is an example file for the interpolation grid.
13.606 13.606 13.606 13.606 13.606 13.606 13.606 13.606 13.606 13.606 ……  ……  ……  ……  ……  ……13.606 13.606
13.903 13.902 13.901 13.900 13.899 13.897 13.896 13.895 13.894 13.893 ……  ……  ……  ……  ……  ……13.902 13.903
……
……
5.502 4.969 4.279 3.243 1.921 0.616 -0.409 -1.156 -1.807 -2.470 ……  ……  ……  ……  ……  ……5.501 5.502
……
……
-29.547 -29.548 -29.549 -29.550 -29.551 -29.552 -29.553 -29.554 -29.555 -29.556 ……  ……  ……  ……  ……  ……-29.546 -29.547
-29.534 -29.534 -29.534 -29.534 -29.534 -29.534 -29.534 -29.534 -29.534 -29.534 ……  ……  ……  ……  ……  ……-29.534 -29.534
```